Analysis of resonant earthed networks

This document has been written to support a joint study into theoretical modelling of resonant earthing commissioned by the four electricity distribution businesses in Victoria Australia in 2014. The goal of the study was to better understand the transient behaviour of resonant earthed networks to gain insights into the fire risk associated with powerline earth faults in such networks.

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Purpose of the analysis

Resonant earthed systems are electricity distribution networks that are earthed by the connection of an inductor (an Arc Suppression Coil or ASC) between the neutral connection of the zone substation transformers and the zone substation earth grid.

There are many published formulae that predict the steady state value of residual conductor voltage and current for an earth fault in a resonant earthed network, but few that accurately predict the transition from pre-fault to post-fault steady state conditions or the transition that occurs when the fault current ceases to flow. Calculation methods for these transitions are potentially important to assessment of fire risk in earth faults as they predict how fast the voltage on the faulted phase will collapse (i.e. how soon arcs might self-extinguish) and how fast it will rise again if compensation is switched off.

The following analysis relies on three simplifications:

1. It uses a simplified 50Hz lumped element equivalent circuit – it doesn't address higher frequency effects in the first 20 milliseconds that involve travelling waves that require distributed element models of powerlines and substation components. The transients of interest (those that follow occurrence and removal of a fault) take place over hundreds of milliseconds, whereas the initial high frequency transient is usually gone in 20 milliseconds.
2. The earth fault is assumed to be a constant linear resistance connecting one phase to earth, so the system equation is linear and the principle of superposition can be used to simplify the analysis considerably. A more realistic non-linear fault resistance would create major complexity in the mathematics and may be best handled by time domain simulation.
3. Unless otherwise stated, the network is perfectly balanced: the capacitance and leakage resistance of each phase to ground is equal to that on the other two phases and the three phase voltages are perfectly balanced at the source substation. This allows the whole array of balanced three-phase voltages and currents to be subtracted from the full representation of the system, to leave a single phase equivalent circuit that determines the neutral displacement voltage. There is always some imbalance in real networks and additional analysis can address this.

The analysis results have been confirmed using time domain simulation of network models that match the parameters used in the analysis. More complex distributed-element network models have confirmed that the theory derived from lumped-element analysis still holds in that such models produce results sets that are internally self-consistent with the theory.

The results have been compared with field test results only in general terms to date.

The author would like to acknowledge the published outline *Oscillations and Waves* by Professor Richard Fitzpatrick of the University of Texas at Austin. This short treatise was useful confirmation of a suitable high level structure for theoretical analysis of forced harmonic oscillators, of which this document is one example.

Outline of the analysis

The analysis presented here is best understood as the following sequence of broadly defined tasks:

1. Establishment of the basic analysis framework using an un-faulted network
    a. Derivation of a system equation for a network without an earth fault present
    b. Understanding the equivalent circuit for the neutral voltage
    c. Derivation of steady-state amplitude of neutral voltage from the equivalent circuit
    d. Definition of the form of the transient solution – decay time and frequency
2. Development of a full solution for a network with an earth fault
    a. Exploration of the changes that an earth fault causes in the equations
    b. Exploration of the changes that an earth fault causes in the neutral equivalent circuit
    c. Exploration of the behaviour of resonant earthed networks with earth faults
    d. Derivation of a general solution for neutral voltage with an earth fault
        i. Transient solution
        ii. Transient solution for low resistance faults
        iii. Steady-state solution
    e. Definition of the solution for particular initial conditions
        i. Fault occurrence at voltage zero-crossing
        ii. Fault occurrence at voltage peak
3. Exploration of the effect of network imbalance on the neutral voltage
4. Investigation of back-fed 'wire down' faults
    a. Fault detection sensitivity variation with neutral voltage threshold setting
    b. Fault detection sensitivity comparison of resonant earthing and solid earthing
5. Comparison of analysis results with time domain simulation results
6. Investigation of the effect of open-delta regulators on resonant earthed networks
    a. Fault detection through an open-delta regulator
    b. Generation of standing neutral voltage by open-delta regulators

There are two patterns that have emerged in this overall analysis program of work:

First, the basic analysis framework developed for an un-faulted network applies in every subsequent analysis. The same forms of solutions (transient and steady-state) continue to appear and each new situation simply produces adjusted values for the key parameters of decay time constant, network damping resistance, transient solution frequency, etc. This means that a thorough understanding of the analysis of the un-faulted network provides the basis for all subsequent results.

Second, the derivation of a solution for each new network configuration follows the same process:

- Develop the circuit diagram for the configuration under investigation
- List all the voltage and current equations
- Apply Kirchhoff's Current Law to the neutral connection point
- Use the voltage and current equations to reduce the resulting equation to functions of time and neutral voltage only
- Compare the resultant neutral voltage equation to the basic one for the un-faulted network to infer the transient and steady-state solutions for this configuration.

This indicates the overall method set out in this document can be used to deal with future challenges, not just the ones covered here.

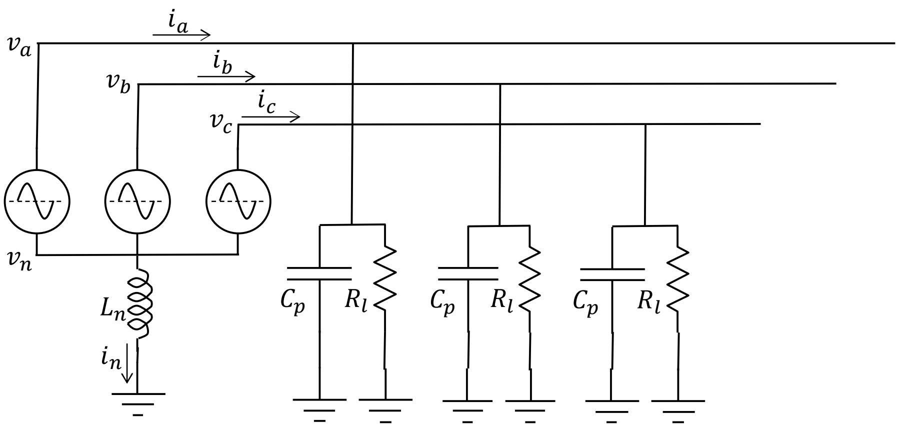
Neutral voltage with no earth fault

As a first step it is worth developing a mathematical model of a resonant-earthed balanced three-phase network in steady state with no earth fault.

Equivalent circuit for phase voltages

After the simplifying assumptions listed above, the system looks like Figure 1:

Figure 1: balanced resonant network



The various parameters are:

L_n is the inductance of the arc suppression coil

C_p is the capacitance to earth of each phase of the network

R_l is the leakage resistance to earth of each phase of the network.

Voltages

The voltages of the three phase conductors with respect to earth will be equal to the neutral voltage plus a three-phase set of balanced voltages:

$$v_a(t) = v_n(t) + V_p \sin(\omega t) \quad (1)$$

$$v_b(t) = v_n(t) + V_p \sin\left(\omega t + \frac{2\pi}{3}\right) \quad (2)$$

$$v_c(t) = v_n(t) + V_p \sin\left(\omega t + \frac{4\pi}{3}\right) \quad (3)$$

Adding these three equations immediately yields:

$$v_n(t) = \frac{1}{3}[v_a(t) + v_b(t) + v_c(t)] \qquad (4)$$

Equation (4) was used in the Victoria's 2014 REFCL Trial to derive the neutral displacement from field test records of the three phase voltages. It has also proven very useful in the following analysis.

Currents

The currents that will flow in each phase conductor will arise from the capacitance between each conductor and earth and the leakage currents between each conductor and earth:

$$i_a(t) = \frac{v_n(t)}{R_l} + C_p\frac{dv_n(t)}{dt} + \frac{V_p}{R_l}sin(\omega t) + V_p\omega C_p\,cos(\omega t) \qquad (5)$$

$$i_b(t) = \frac{v_n(t)}{R_l} + C_p\frac{dv_n(t)}{dt} + \frac{V_p}{R_l}sin\left(\omega t + \frac{2\pi}{3}\right) + V_p\omega C_p\,cos\left(\omega t + \frac{2\pi}{3}\right) \qquad (6)$$

$$i_c(t) = \frac{v_n(t)}{R_l} + C_p\frac{dv_n(t)}{dt} + \frac{V_p}{R_l}sin\left(\omega t + \frac{4\pi}{3}\right) + V_p\omega C_p\,cos\left(\omega t + \frac{4\pi}{3}\right) \qquad (7)$$

Since the neutral connection point is earthed through an inductor, L_n, then:

$$i_n(t) = \frac{1}{L_n}\int v_n(t)\;dt \qquad (8)$$

System equation

Application of Kirchhoff's Current Law to the neutral connection point gives:

$$0 = i_n(t) + i_a(t) + i_b(t) + i_c(t) \qquad (9)$$

So the system equation for the neutral voltage (after all the three-phase balanced sets of terms sum to zero) is:

$$0 = \frac{1}{L_n}\int v_n(t)\;dt + \frac{3}{R_l}v_n(t) + 3C_p\frac{dv_n(t)}{dt} \qquad (10)$$

This second order linear differential equation for $v_n(t)$ describes a resonant parallel RLC circuit. It is more usually described (after differentiating and dividing through by C_p) as:

$$0 = \frac{1}{C_nL_n}v_n(t) + \frac{1}{C_nR_n}\frac{dv_n(t)}{dt} + \frac{d^2v_n(t)}{dt^2} \qquad (11)$$

Where C_n and R_n are total network parameters:

$$Total\ network\ damping\ resistance: R_n = \frac{R_l}{3} \quad (12)$$

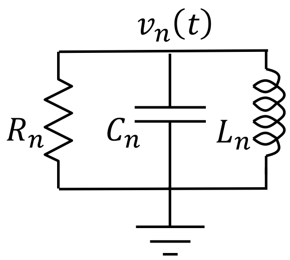
$$Total\ nework\ capacitance: C_n = 3C_p \quad (13)$$

In real networks, R_n is slightly less than the value implied by per-phase leakage currents as the associated resistive current must supply other energy losses in the network, some of which are due to the series resistance of phase conductors and iron losses in the resonant inductor coil, etc. However, since R_l can never be directly measured, only R_n is of meaningful interest – it can be deduced from earth fault test results.

Equivalent circuit for neutral voltage

Solutions to the system equation (11) are useful when seeking to understand the transient response of the network when an earth fault is not present, e.g. when it is removed. Equation (11) describes the circuit shown in Figure 2:

Figure 2: equivalent circuit for neutral voltage



It can be seen there is no energy source in this system and any non-zero value of $v_n(t)$ will create energy loss through the current it causes to flow in resistor R_n. A trivial solution for Equation (11) is: $v_n(t) = 0$, i.e. no neutral voltage displacement. As this particular solution sets the total energy of the system at zero, it will be the solution to which the system will naturally converge in the absence of any stimulus.

Transient response of an un-faulted network

In most networks, R_n is relatively large (i.e. energy leakage is low) so it is easy to postulate that the sudden application of a disturbance to the resonant system at time zero will create a transient oscillatory response which eventually decays away again to the stable solution $v_n(t) = 0$. In such an under-damped system, we can consider a response in the general form of a decaying sinusoid:

$$v_n(t) = V_{nt}e^{-\gamma t}\sin(\omega_1 t + \varphi_t) \quad (14)$$

The first derivative of $v_n(t)$ is:

$$\frac{dv_n(t)}{dt} = V_{nt}e^{-\gamma t}\omega_1\cos(\omega_1 t + \varphi_t) - V_{nt}e^{-\gamma t}\gamma\sin(\omega_1 t + \varphi_t) \quad (15)$$

The second derivative of $v_n(t)$ is:

$$\frac{d^2 v_n(t)}{dt^2} = V_{nt} e^{-\gamma t} \gamma^2 \sin(\omega_1 t + \varphi_t) - 2V_{nt} e^{-\gamma t} \gamma \omega_1 \cos(\omega_1 t + \varphi_t) - V_{nt} e^{-\gamma t} {\omega_1}^2 \sin(\omega_1 t + \varphi_t) \quad (16)$$

These can be substituted in Equation (11) which can then be described as having:

- $V_{nt} e^{-\gamma t}$ in all terms - so this expression can be divided out of the whole equation
- Two terms containing $\cos(\omega_1 t + \varphi_t)$ and three terms containing $\sin(\omega_1 t + \varphi_t)$.

For Equation (11) to be satisfied at all times, the $\cos(\omega_1 t + \varphi_t)$ terms and the $\sin(\omega_1 t + \varphi_t)$ terms must separately sum to zero. The two conditions to be met for this to happen are:

$$0 = \frac{1}{L_n} - \frac{\gamma}{R_n} - \omega_1^2 C_n + \gamma^2 C_n \quad (17)$$

$$0 = \frac{\omega_1}{R_n} - 2\gamma \omega_1 C_n \quad (18)$$

Equation (18) immediately gives us a value for the decay time constant of the transient solution and equation (17) then gives us the frequency of the transient solution:

$$\gamma = \frac{1}{2R_n C_n} \; so\ the\ decay\ time\ constant\ \tau_n = \frac{1}{\gamma} = 2R_n C_n \quad (19)$$

$$\omega_1 = \omega_0 \sqrt{(1 - \zeta^2)} \quad (20)$$

Where damping factor $\zeta = \frac{\gamma}{\omega_0}$ and system resonant frequency $\omega_0 = \frac{1}{\sqrt{L_n C_n}}$.

It can be seen that:

1. The damping in the system (represented by a finite value for R_n) reduces the frequency of the transient response below the resonant frequency of the system, i.e. $\omega_1 < \omega_0$.
2. The decaying sinusoid form of solution is not valid for $\zeta > 1$ (as R_n decreases, the frequency first falls to zero and then becomes imaginary) and a non-oscillatory exponential decay transient is likely to be a more valid solution.
3. If there is no damping ($R_n = \infty$), the transient response never dies away ($\gamma = 0$) and the frequency of the transient response is the system resonant frequency ($\omega_1 = \omega_0$).

The transient solution still has two degrees of freedom (V_{nt}, φ_t) which can be determined by initial conditions, using equations (14), (15) and (16).

This solution will apply following removal of an earth fault. The form of the transient will depend upon whether the neutral voltage or the ASC coil current (or both) are non-zero at the instant of fault removal. For example, at time $\mathrm{t} = 0$ let $v_n(0) = v_{n0}$ and $i_n(0) = i_{n0}$. Equations (14), (10) and (15) then give:

$$\frac{v_{n0}}{V_{nt}} = sin(\varphi_t) \tag{21}$$

$$\frac{i_{n0}}{V_{nt}} = -\left[\frac{1}{R_n} + \gamma C_n\right] sin(\varphi_t) + C_n\omega_1\, cos(\varphi_t) \tag{22}$$

$$i_{n0} = -\left[\frac{1}{R_n} + \gamma C_n\right] v_{n0} + C_n\omega_1\sqrt{1 - \left(\frac{v_{n0}}{V_{nt}}\right)^2} \tag{23}$$

This leads to the following solution for the two variables, V_{nt} and φ_t:

$$V_{nt} = \frac{v_{n0}}{\sqrt{1 - \left\{\frac{i_{n0} + \left[\frac{1}{R_n} + \gamma C_n\right] v_{n0}}{C_n\omega_1}\right\}^2}}$$ and then: $\varphi_t = sin^{-1}\frac{v_{n0}}{V_{nt}}$

With a fault present, these two variables in the transient solution must be determined from initial conditions in a similar way, but this time using the full general solution as shown in a later section.

Example: transient response of test system

At the zone substation supplying the field test site, the parameters shown in Table 1 were found to apply:

Table 1: Network parameters at zone substation supplying the field test site

Parameter	Formula	Value
ASC inductance	$L_n = \frac{12.7kV}{2\pi f\, ASC_{pos}} = \frac{12700}{314\ \ 144}$	0.281 H
Network capacitance	$C_n = \frac{1}{(2\pi f)^2 L_n} = \frac{1}{314^2\ 0.281}$	36.1 µF
Network resistance	Deduced from GFN tuning data	4,000 Ω

ASC_{pos} is the operating position of the ASC coil at resonance, expressed in amps. Applying Equations (19) and (20), the characteristics of transients in the neutral voltage with no faults on the network are as shown in Table 2:

Table 2: test network transient parameters with no earth fault

Parameter	Formula	Value
Transient decay time constant	$\tau_n = 2R_nC_n$ and $\frac{1}{\gamma} = \tau_n$	0.29 seconds ($\gamma = 3.45\ s^{-1}$)
Transient frequency	$\zeta = \frac{\gamma}{\omega_0}$ and $\omega_1 = \omega_0\sqrt{(1-\zeta^2)}$	314.14 radians/s ($\zeta = 0.011$)

It can be seen that with no earth fault on the system, the decay rate of any oscillatory transient in the neutral voltage is relatively slow – it would take about two seconds (seven time constants) to die away to one thousandth of its original magnitude. Also, the frequency of any neutral voltage transient is very close to the resonant frequency of the system – only 0.003Hz different to 50Hz. It would take more than a minute for the phase of the transient response waveform to drift 90°with respect to the network 50Hz waveforms. Given the transient lasts only two seconds, the frequency difference between ω_1 and ω_0 is unlikely to materially affect the transient response.

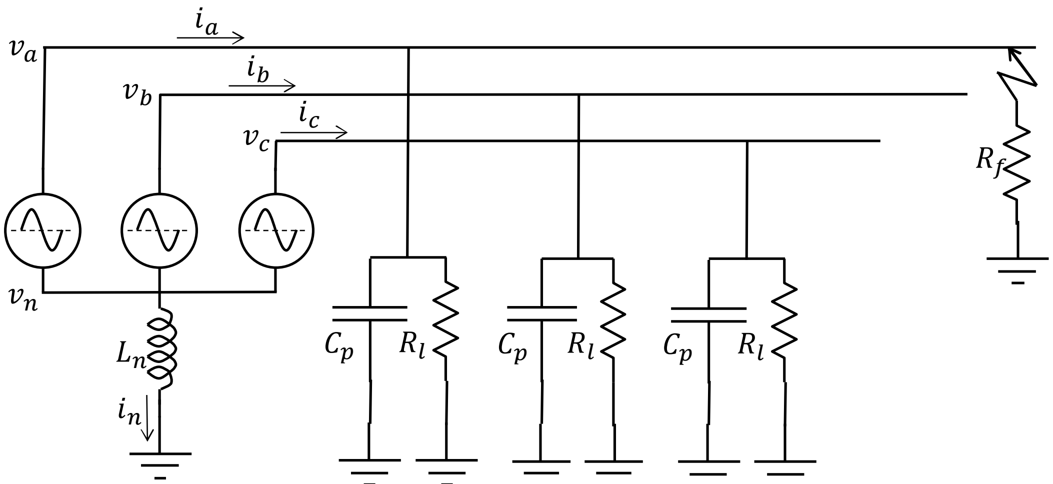
Neutral voltage with an earth fault

With an earth fault on Phase A, the same mathematical modelling can be done as has been done above for the case without any earth fault.

Equivalent circuit for phase voltages with an earth fault

With a fault consisting of a constant linear fault resistance, R_f, on Phase A the network is now as shown in Figure 3:

Figure 3: balanced resonant network with earth fault on Phase A



Voltages with an earth fault

The presence of the fault does not change the voltage equations as there is no series impedance in any of the phases of the network, i.e. Equations (1) to (3) above still apply.

Currents with an earth fault

The current that flows in the Phase A conductor will now have two extra terms due to fault current:

$$i_a(t) = \frac{v_n(t)}{R_l} + C_p \frac{dv_n(t)}{dt} + \frac{V_p}{R_l} sin(\omega t) + V_p \omega C_p \, cos(\omega t) + \frac{v_n(t)}{R_f} + \frac{V_p}{R_f} sin(\omega t) \quad (24)$$

The currents in the other two phases are unchanged, i.e. Equations (6) and (7) apply. The neutral is still earthed through the arc suppression coil inductance, L_n, so Equation (8) also still applies.

System equation with an earth fault

Applying Kirchhoff's Current Law to the neutral connection point using Equations (24), (6), (7), and (8), the system equation for the neutral voltage with an earth fault is:

$$-\frac{V_p}{R_f} sin(\omega t) = \frac{1}{L_n} \int v_n(t)\ dt + \left[\frac{1}{R_f} + \frac{1}{R_n}\right] v_n(t) + C_n \frac{dv_n(t)}{dt} \quad (25)$$

Differentiating and dividing through by C_n gives:

$$-\frac{\omega V_p}{R_f C_n} cos(\omega t) = \frac{1}{C_n L_n} v_n(t) + \frac{1}{R_{eq} C_n}\frac{dv_n(t)}{dt} + \frac{d^2 v_n(t)}{dt^2} \quad (26)$$

Where R_{eq} is an equivalent network leakage resistance to earth, equal to the resistance of the parallel combination of the fault resistance and the total network leakage resistance:

$$R_{eq} = \frac{R_n R_f}{R_n + R_f} \quad (27)$$

Comparing Equation (26) to Equation (11) , it can be seen the presence of the earth fault changes the damping of the resonant system and adds a non-zero term on the left hand side. This term is called a forcing function because it represents injected energy which 'forces' $v_n(t)$ to adopt a non-zero solution.

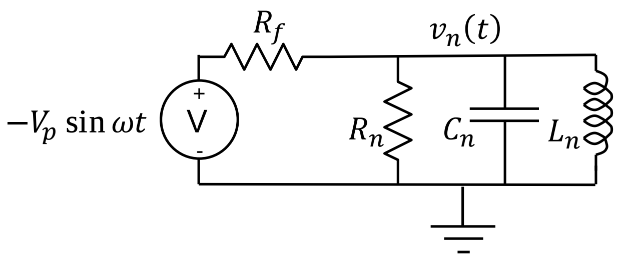
Equivalent circuit for neutral voltage with an earth fault

Re-arranging Equation (25) slightly gives:

$$-\frac{V_p}{R_f} \sin(\omega t) - \frac{1}{R_f} v_n(t) = \frac{1}{L_n} \int v_n(t)\ dt + \frac{1}{R_n} v_n(t) + C_n \frac{dv_n(t)}{dt} \quad (28)$$

This second order linear differential equation for $v_n(t)$ describes a resonant parallel RLC circuit driven by a sinusoidal voltage source equal to the Phase A voltage acting through a resistance equal to the fault resistance, i.e. Equation (28) describes the circuit shown in Figure 4:

Figure 4: equivalent circuit for neutral voltage with an earth fault

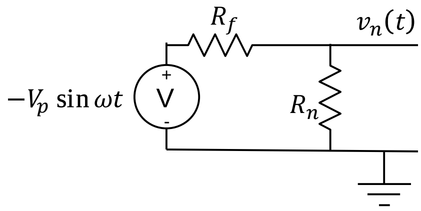


Behaviour of resonant-earthed network with an earth fault

A complete analysis of the general solution for $v_n(t)$ is set out in following sections, but there is an easy way to deduce the steady-state value of $v_n(t)$ from the equivalent circuit shown in Figure 4.

As the neutral inductor L_n is assumed to be perfectly tuned to the network capacitance C_n at 50Hz, once any transient effects have died away the parallel combination of these two components will act as an open circuit (their 50Hz admittances cancel to zero) and the equivalent circuit for the neutral voltage is as shown in Figure 5:

Figure 5: equivalent circuit for neutral voltage in a perfectly tuned network with an earth fault



So the neutral voltage, calculating the effect of the resistive voltage divider, is:

$$v_n(t) = -\frac{R_n}{(R_n+R_f)} V_p \sin(\omega t) \tag{29}$$

Using Equation (1), the voltage on the faulted conductor is:

$$v_a(t) = \frac{R_f}{(R_n+R_f)} V_p \sin(\omega t) \tag{30}$$

This simple equivalent circuit gives sensible results as $R_f \to \infty$ and as $R_f \to 0$, i.e. no fault and short circuit fault as shown in Table 3:

Table 3: network with no fault and with short circuit earth fault

Circumstance:	No fault on network	Short circuit on network
$\boldsymbol{R_f}$	$R_f \to \infty$	$R_f \to 0$
$\boldsymbol{v_n(t)}$	$v_n(t) \to 0$	$v_n(t) \to -V_p \sin(\omega t)$
$\boldsymbol{R_{eq}}$	$R_{eq} \to R_n$	$R_{eq} \to R_f \to 0$
Result:	No neutral displacement	100% neutral displacement

If $R_f = 0$, i.e. the earth fault is a short circuit, then Equations (1) and (24) become:

$$v_a(t) = 0 \text{ and } i_a(t) = \frac{0}{0} \tag{31}$$

Equation (31) gives an indeterminate result for the fault current $i_a(t)$, so it must be calculated instead using Kirchhoff's current law as re-expressed in Equation (32).

$$i_a(t) = -i_n(t) - i_b(t) - i_c(t) \tag{32}$$

This gives:

$$\begin{aligned} i_a(t) &= \frac{V_p}{L_n} \int sin(\omega t) - [i_b(t) + i_c(t)] \\ i_a(t) &= K - \frac{V_p}{\omega L_n} cos(\omega t) - [i_b(t) + i_c(t)] \end{aligned} \tag{33}$$

where K is an integration constant. The currents in Phases B and C are:

$$i_b(t) = -\frac{V_p}{R_l} sin(\omega t) - \omega C_p V_p \, cos(\omega t) + \frac{V_p}{R_l} sin\left(\omega t + \frac{2\pi}{3}\right) + V_p \omega C_p \, cos\left(\omega t + \frac{2\pi}{3}\right) \quad (34)$$

$$i_c(t) = -\frac{V_p}{R_l} sin(\omega t) - \omega C_p V_p \, cos(\omega t) + \frac{V_p}{R_l} sin\left(\omega t + \frac{4\pi}{3}\right) + V_p \omega C_p \, cos\left(\omega t + \frac{4\pi}{3}\right) \quad (35)$$

It is useful to use:

$$0 = sin(\omega t) + sin\left(\omega t + \frac{2\pi}{3}\right) + sin\left(\omega t + \frac{4\pi}{3}\right)$$

so: $-\, sin(\omega t) = sin\left(\omega t + \frac{2\pi}{3}\right) + sin\left(\omega t + \frac{4\pi}{3}\right)$ (36)

and similarly: $-\, cos(\omega t) = cos\left(\omega t + \frac{2\pi}{3}\right) + cos\left(\omega t + \frac{4\pi}{3}\right)$

Using these identities in Equations (34) and (35) gives:

$$i_b(t) + i_c(t) = -\frac{3\,V_p}{R_l}\sin(\omega t) - 3\omega C_p V_p \cos(\omega t)$$

So from Equation (33), we get:

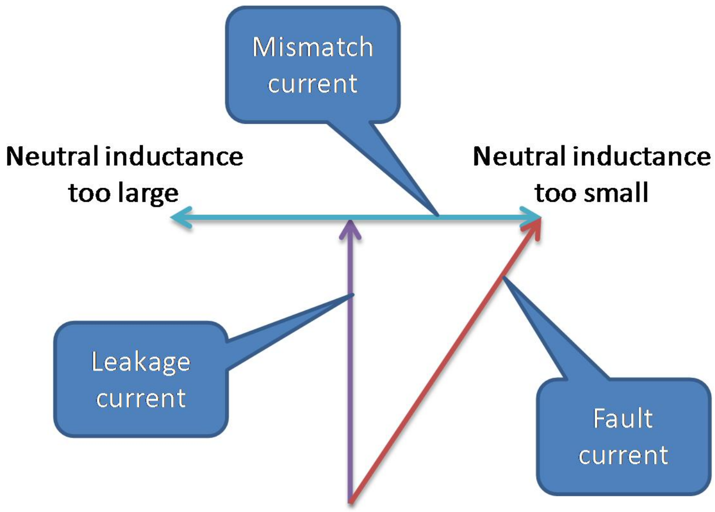
$$i_a(t) = K + V_p\left[\omega C_n - \frac{1}{\omega L_n}\right] cos(\omega t) + \frac{V_p}{R_n} sin(\omega t) \quad (37)$$

In the steady state, $i_a(t)$ will have zero DC component, so the integration constant K can be taken to be zero.

$$i_a(t) = V_p\left[\omega C_n - \frac{1}{\omega L_n}\right] cos(\omega t) + \frac{V_p}{R_n} sin(\omega t) \quad (38)$$

The first term in Equation (38) is 'mismatch current' due to any tuning mismatch between the neutral inductor and the network capacitance. The second is network leakage current to earth. The two are related as shown in Figure 6.

Figure 6: Vector addition of mismatch and leakage currents to form fault current



If the neutral inductor is perfectly tuned to the network, $\frac{1}{\omega L_n} = \omega C_n$, the earth fault current is:

$$i_a(t) = \frac{V_p}{R_n} sin(\omega t) \tag{39}$$

The earth fault behaviour of a resonant-earthed network is summarised in Table 4:

Table 4: phase voltage and short circuit earth fault current in a resonant network

Circumstance:	No fault	Short circuit fault
Conductor voltage	$v_a(t) = V_p \sin(\omega t)$	Zero
Fault current	Zero	$i_a(t) = \frac{V_p}{R_n}\sin(\omega t)$

This confirms a perfectly tuned balanced network exhibits the desired behaviour of a resonant-earthed network: earth fault current is limited to a value equal to the resistive leakage current of the network to earth.

Residual conductor voltage and fault current

The values of neutral displacement, phase voltage and fault current can also easily be found for values of fault resistance that lie between open circuit and short circuit.

The voltage on the faulted phase is given by Equation (30). The fault current reduces in the same proportion as the conductor voltage since the fault resistance is linear:

$$v_a(t) = \frac{R_f}{(R_n+R_f)} V_p \sin(\omega t) \tag{40}$$

$$i_a(t) = \frac{v_a(t)}{R_f} = \frac{1}{(R_n+R_f)} V_p \sin(\omega t) \tag{41}$$

Deduction of network parameters: FSH test data

This offers a way to confirm the total network leakage resistance, R_n. If a 'bolted' earth-fault test using a known fault resistance reveals the steady-state reduction in conductor voltage, then the total network leakage resistance can be checked.

The 2014 REFCL Trial included two series of 'bolted' fault tests in which the white phase was connected to earth eight kilometres from the zone substation through a high voltage series resistor which could be set at resistance values from 100 ohms to 15,200 ohms.

Any standing neutral displacement due to network imbalance must be taken into account in the calculation. Standing neutral displacement is due to imbalance of network parameters (usually capacitance) across the three phases or in the three phase set of source voltages. The analysis of the 2014 test records required vector subtraction of the standing neutral displacement voltage and is not set out here.

Transient and steady-state solutions for a faulted network

Because the system equation is linear, the general solution for the neutral voltage of a network in the presence of an earth fault on one phase can be derived using superposition.

Using superposition to get a general solution

The general solution of Equation (26) can be derived as the sum of two parts:

1. A transient solution of the equation:

$$0 = \frac{1}{C_n L_n} v_n(t) + \frac{1}{R_{eq} C_n} \frac{dv_n(t)}{dt} + \frac{d^2 v_n(t)}{dt^2} \qquad (42)$$

   Because the transient solution dies away to zero, there can be no forcing function. A forcing function would imply ongoing energy injection so the solution cannot die away to zero, i.e. a solution with a forcing function cannot be a transient solution.

2. A steady state solution of the equation:

$$-\frac{\omega V_p}{R_f C_n} cos(\omega t) = \frac{1}{C_n L_n} v_n(t) + \frac{1}{R_{eq} C_n} \frac{dv_n(t)}{dt} + \frac{d^2 v_n(t)}{dt^2} \qquad (43)$$

   Since the forcing function is of constant amplitude over time, the response to it should also be of constant amplitude at the same frequency as the forcing function.

Each of these solutions can be separately derived.

Transient response of faulted network

Comparing Equation (42) with Equation (11) and the transient solution given by Equations (14), (19) and (20), it can be seen that the transient solution of a network with an earth fault is of the same form as that for the un-faulted network but with modified damping. The presence of the fault changes both the decay time constant and the frequency of the transient. In effect the fault resistance appears in parallel with the leakage resistance to define a new damping resistance, R_{eq}.

Example: transient response - test system with medium resistance earth fault

With the 22kV test network parameters set out in Table 1, assume a 240 ohm earth fault – about equivalent to a fifty amp earth fault if the network were solidly earthed. The key parameters in the network transient response to the earth fault are set out in Table 5:

Table 5: test network response to a 240 ohm earth fault

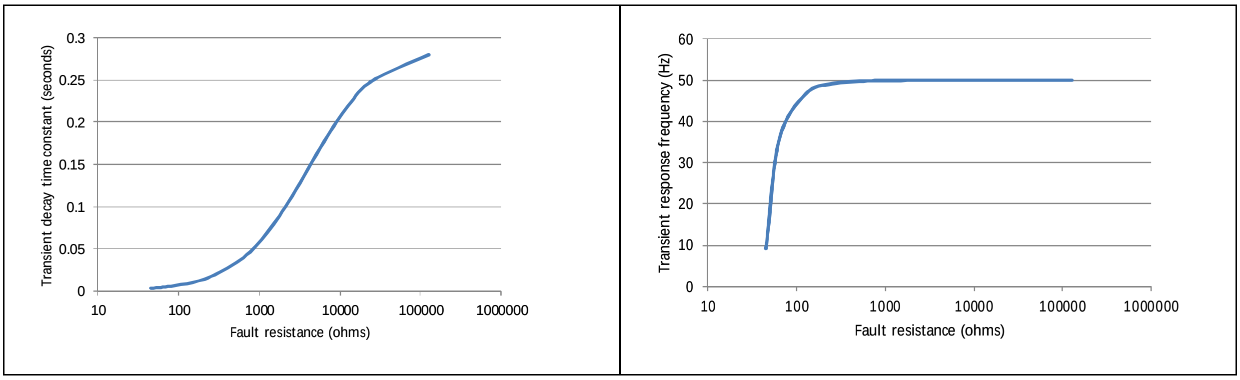
Parameter	Formula	Value
Fault resistance	Hypothesised for fifty amp fault	250 Ω
Equivalent network resistance	$R_{eq} = \frac{R_n R_f}{R_n + R_f}$	235 Ω
Transient decay time constant	$\tau_n = 2R_{eq} C_n = \frac{1}{\gamma}$	0.017 seconds (γ = 59 s^{-1})
Transient frequency	$\zeta = \frac{\gamma}{\omega_0}$ and $\omega_1 = \omega_0 \sqrt{(1 - \zeta^2)}$	308.6 radians/s (49.11Hz)

The decay is very rapid with the transient diminishing to 0.1% of its initial value in just 120ms. Also, the frequency of the transient is 0.9Hz away from the system frequency, so there may be a very slight 'beating' effect in the combination of the transient and steady-state elements of the solution, perhaps enough to move the envelope of the initial transient away from an exact exponential curve.

The network decay time constant and the transient solution frequency vary with fault resistance as shown in Figure 7 which demonstrates that:

1. At lower fault resistance values, the decay time constant is inversely proportional to fault resistance. For a 100 ohm fault, it is only nine milliseconds.
2. At higher values of fault resistance, the decay time constant approaches the value that applies in an un-faulted network.
3. The frequency of the transient response stays close to 50Hz until the fault resistance reduces below about 100 ohms.
4. Generally, the transient response has died away to zero well before a single cycle of 'beat' frequency between the transient response frequency and the 50Hz network frequency. For a fault resistance of fifty ohms, the transient response has a frequency of 22.5Hz and a decay time constant of 3.6ms. The transient response would have diminished to 0.1% of its initial value in just 0.7 of a cycle of 'beat' frequency.

Figure 7: variation of decay time constant and transient response frequency with fault resistance



This implies that any visible 'beating' in transient response waveforms is more likely due to tuning mismatch than the effect of the fault resistance on the transient response frequency.

Transient solution for low resistance faults

If R_f is very small, the transient response cannot have an oscillatory character since $\zeta > 1$ in Equation (20) and the oscillation frequency ω_1 of the transient response would be imaginary. The threshold fault resistance at which this boundary is crossed is given by:

$$\gamma = \omega_0, i.e. \frac{1}{2R_{eq}C_n} = \omega_0 \text{ which is: } R_{eq} = \frac{1}{2\omega_0 C_n} \; or \; \frac{R_f}{\left(1 + \frac{R_f}{R_n}\right)} = \frac{1}{2\omega_0 C_n} \; or$$
$$R_f = \frac{1}{2\omega_0 C_n} + \frac{R_f}{2\omega_0 C_n R_n} \text{ which is: } R_f \left[1 - \frac{1}{2\omega_0 C_n R_n}\right] = \left[\frac{1}{2\omega_0 C_n}\right] \qquad (44)$$

This gives the threshold value of R_f:

$$R_f == \frac{R_n}{(2\omega_0 C_n R_n - 1)} \quad (45)$$

For the test network parameters listed in Table 1, the threshold value of earth fault resistance is $R_f = 44.6$ ohms, representing a fault that in a solidly earthed network would draw 285 amps. In a resonant earthed network, faults with resistance less than this value have no oscillatory component in the transient response.

For low-resistance faults, we should seek a solution for $v_n(t)$ of the form:

$$v_n(t) = V_{nt} e^{-\propto t} \quad (46)$$

$$\frac{dv_n(t)}{dt} = -V_{nt} \propto e^{-\propto t} \quad (47)$$

$$\frac{d^2 v_n(t)}{dt^2} = V_{nt} \propto^2 e^{-\propto t} \quad (48)$$

And Equation (42) becomes:

$$0 = \frac{1}{C_n L_n} V_{nt} e^{-\propto t} - \frac{V_{nt}}{R_{eq} C_n} \gamma e^{-\propto t} + V_{nt} \propto^2 e^{-\propto t} \quad (49)$$

Dividing by $V_{nt} e^{-\propto t}$, this is a quadratic equation in $\propto$:

$$0 = \frac{1}{C_n L_n} - \frac{1}{R_{eq} C_n} \propto + \propto^2 \quad \text{which is:} \quad \propto^2 - 2\gamma \propto + \omega_0^2 = 0 \quad (50)$$

This has the solution:

$$\propto = \gamma \left[1 \pm \frac{1}{\zeta} \sqrt{(\zeta^2 - 1)}\right] \text{ where } \zeta = \frac{\gamma}{\omega_0} \quad (51)$$

Equation (51) has two valid solutions for $\propto$, so the transient solution can be the sum of two exponential decay components:

$$v_n(t) = V_{nt1} e^{-\propto_1 t} + V_{nt2} e^{-\propto_2 t} \quad (52)$$

Where:

$$\propto_1 = \gamma \left[1 + \frac{1}{\zeta} \sqrt{(\zeta^2 - 1)}\right] \quad (53)$$

$$\propto_2 = \gamma \left[1 - \frac{1}{\zeta} \sqrt{(\zeta^2 - 1)}\right] \quad (54)$$

Equation (46) has two parameters V_{nt1} and V_{nt2} which can be determined to suit initial conditions when the general solution is fully defined.

Example: transient response of test system with low resistance earth fault

For the test network parameters set out in Table 1, assume a two ohm earth fault – about equivalent to a 1.3 kilo-amp earth fault if the network were earthed via an eight ohm NER (i.e. total earth fault resistance is ten ohms). The key parameters in the network transient response to the earth fault are set out in Table 5:

Table 6: test network response to a 10 ohm earth fault

Parameter	Formula	Value
Fault resistance	Hypothesised	10 Ω
Equivalent network resistance	$R_{eq} = \frac{R_n R_f}{R_n + R_f}$	9.975 Ω
Damping factor	$\zeta = \frac{\gamma}{\omega_0} = \frac{1}{2R_{eq}}\sqrt{\frac{L_n}{C_n}}$	4.42
Transient time constant 1	$\tau_n = \frac{1}{\propto_1}$	364 microseconds
Transient time constant 2	$\tau_n = \frac{1}{\propto_2}$	27.9 milliseconds

This example demonstrates that very low resistance faults (such as a high voltage conductor falling onto low voltage subsidiary spans that are solidly earthed through multiple earthed neutral connections in connected premises) can generate very fast transients. As fault resistance reduces, the two time constants separate. One approaches zero and one approaches $\frac{1}{\mathrm{R_{eq}C_n}}$ (the value of which is also reducing as the fault resistance reduces).

Steady state solution of faulted network

After the transient component of $\mathrm{v_n(t)}$ has died away, it will consist only of the steady-state solution. It is reasonable to look for a steady-state solution for Equation (43) of the form:

$$v_n(t) = V_{ns}\, cos(\omega t + \varphi_s) \quad (55)$$

The first derivative of $v_n(t)$ is then:

$$\frac{dv_n(t)}{dt} = -\omega V_{ns}\, sin(\omega t + \varphi_s) \quad (56)$$

and the second derivative of $v_n(t)$ is:

$$\frac{d^2 v_n(t)}{dt^2} = -\omega^2 V_{ns} \cos(\omega t + \varphi_s) \quad (57)$$

The system Equation (43) then becomes:

$$-\frac{\omega V_p}{R_f C_n} cos(\omega t) = \frac{V_{ns}}{C_n L_n} cos(\omega t + \varphi_s) - \frac{\omega V_{ns}}{R_{eq} C_n} sin(\omega t + \varphi_s) - \omega^2 V_{ns}\, cos(\omega t + \varphi_s) \quad (58)$$

This can be rewritten:

$$\frac{\omega V_p}{R_f C_n} cos(\omega t) = 2\gamma\omega V_{ns} sin(\omega t + \varphi_s) + (\omega^2 - \omega_0^2)V_{ns} cos(\omega t + \varphi_s) \quad (59)$$

Where, as before with the earth fault present, the value of the network damping resistance is R_{eq}, so:

$$\omega_0^2 = \frac{1}{\sqrt{L_n C_n}} \text{ and } \gamma = \frac{1}{2R_{eq}C_n} \quad (60)$$

Two trigonometric identities can be used to expand the $(\omega t + \varphi_s)$ sinusoid terms in Equation (59):

$$\sin(\omega t + \varphi_s) \equiv \sin\omega t \cos\varphi_s + \cos\omega t \sin\varphi_s$$

$$\cos(\omega t + \varphi_s) \equiv \cos\omega t \cos\varphi_s - \sin\omega t \sin\varphi_s$$

Using these two identities, we get two equations, one of all the terms that include $\sin\omega t$ and one of all the terms that include $\cos\omega t$. Both have to be separately satisfied for Equation (58) to be satisfied for all instants in time:

$$\frac{\omega V_p}{R_f C_n} = V_{ns}[\omega^2 - \omega_0^2] \cos\varphi_s + 2\gamma\omega V_{ns} \sin\varphi_s \quad (61)$$

$$0 = -V_{ns}[\omega^2 - \omega_0^2] \sin\varphi_s + 2\gamma\omega V_{ns} \cos\varphi_s \quad (62)$$

Equation (62) can be written:

$$\varphi_s = tan^{-1}\left[\frac{2\gamma\omega}{(\omega^2-\omega_0^2)}\right] \quad (63)$$

Again, convenient trigonometric identities can be used to define $\cos\varphi_s$ and $\sin\varphi_s$:

$$\cos\varphi_s \equiv \frac{1}{\sqrt{(1+\tan^2\varphi_s)}} = \frac{(\omega^2-\omega_0^2)}{\sqrt{\left[(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2\right]}}$$

$$\sin\varphi_s \equiv \frac{\tan\varphi_s}{\sqrt{(1+\tan^2\varphi_s)}} = \frac{2\gamma\omega}{\sqrt{\left[(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2\right]}}$$

Substituting the above values of $\cos\varphi$ and $\sin\varphi$ in Equation (61) gives:

$$\frac{\omega V_p}{R_f C_n} = V_{ns}[(\omega^2 - \omega_0^2) \cos\varphi_s + 2\gamma\omega \sin\varphi_s] = V_{ns}\sqrt{[(\omega^2 - \omega_0^2)^2 + 4\gamma^2\omega^2]} \quad (64)$$

Using Equation (19) with R_{eq} substituted for R_n, the amplitude of the steady state resonant response can be written:

$$V_{ns} = \frac{V_p}{R_f C_n}\frac{\omega}{\sqrt{\left[(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2\right]}} = \frac{R_{eq}}{R_f}V_p\frac{2\gamma\omega}{\sqrt{\left[(\omega^2-\omega_0^2)^2+4\gamma^2\omega^2\right]}} = \frac{R_{eq}}{R_f}V_p\frac{1}{\sqrt{\left[\left(\frac{(\omega^2-\omega_0^2)}{2\gamma\omega}\right)^2+1\right]}} \quad (65)$$

Testing this result, it can be seen that for a perfectly tuned system where $\omega = \omega_0$, Equation (65) reduces to:

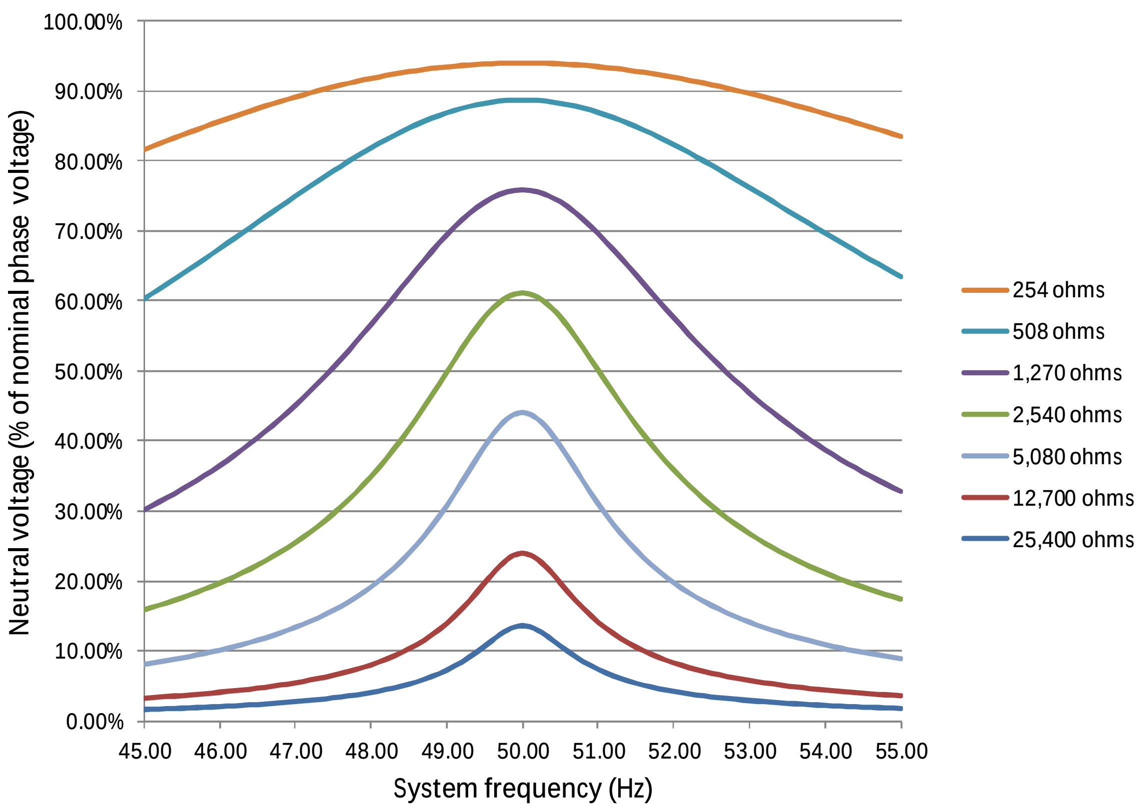
$$V_{ns} = \frac{R_{eq}}{R_f} V_p = \frac{R_n V_p}{R_f + R_n} \quad (66)$$

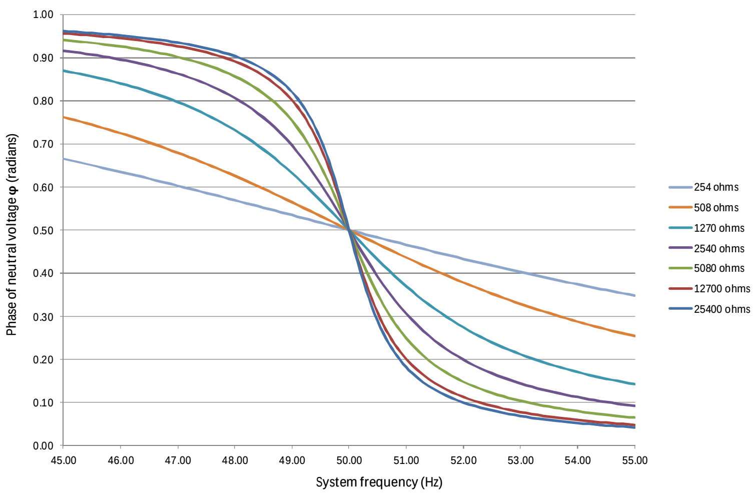
This is the same result as Equation (29).

Example: steady state resonant response of FSH network

Using the test network parameters shown in Table 1, Equations (63) and (65) describe the steady-state resonant response shown in Figure 8.

Figure 8: test network steady-state resonant response (for different values of fault resistance)





At resonance, the phase of the neutral voltage is 90°, i.e. it acts to (partially) cancel the Phase A voltage:

$$v_n(t) = V_{ns}\, cos(\omega t + 90°) = -V_{ns} \sin \omega t = -\left(\frac{R_n}{R_f + R_n}\right) V_p \sin \omega t \qquad (67)$$

$$v_a(t) = v_n(t) + V_p \sin \omega t = V_p \left(1 - \frac{R_{eq}}{R_f}\right) \sin \omega t = \left(\frac{R_f}{R_f + R_n}\right) V_p \sin \omega t \qquad (68)$$

Equation (67) shows that the neutral voltage rise due to a fault depends on the network damping, represented by resistance R_n. Figure 8 shows neutral voltage sensitivity for the test network with a network damping resistance $R_n = 4000\Omega$. Figure 9 shows the test network resonance response with no damping and with heavy damping. It can be seen that the network damping determines the sharpness of the resonance curve.

Figure 9: resonance of test network neutral voltage for no network damping ($R_n = \infty$)

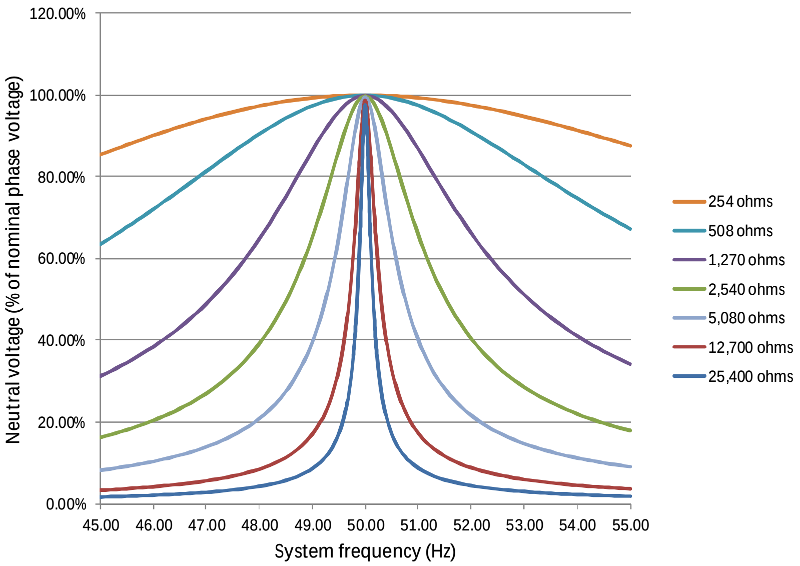
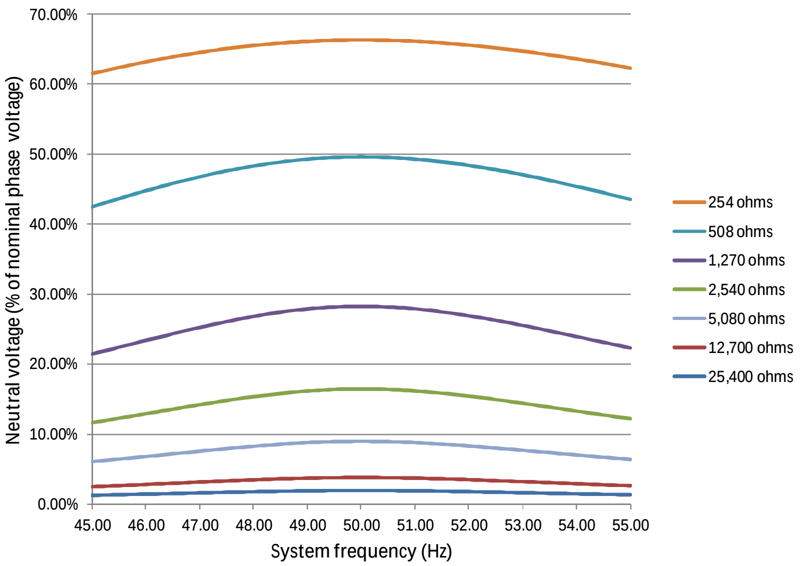


Figure 10: resonance of test network with heavy damping ($R_n = 500\Omega$)



One measure of the sharpness of the resonance curve is the difference between the two frequencies at which the amplitude of V_n is $\frac{1}{\sqrt{2}}$ of the maximum amplitude. These frequencies can be derived from Equation (65) which can be written out and manipulated as follows:

$$\frac{1}{\sqrt{\left[\left(\frac{(\omega^2-\omega_0^2)}{2\gamma\omega}\right)^2+1\right]}} = \frac{1}{\sqrt{2}} \text{ so: } \left(\frac{(\omega^2-\omega_0^2)}{2\gamma\omega}\right)^2 = 1 \text{ so: } \frac{(\omega^2-\omega_0^2)}{2\gamma\omega} = \pm 1 \text{ so: } (\omega^2-\omega_0^2) = \pm 2\gamma\omega$$

$$(\omega^2-\omega_0^2) = (\omega-\omega_0)(\omega+\omega_0) \cong 2\omega_0(\omega-\omega_0) \cong 2\omega(\omega-\omega_0) \; if \; \omega \approx \omega_0$$

So: $(\omega-\omega_0) = \pm\gamma$ or: $\omega = \omega_0 \pm \gamma$

So the width of the resonance curve 30% below the peak value is approximately: $2\gamma = \frac{1}{R_{eq}C_n}$.

General solution for a faulted network

For medium and high fault resistances, the transient solution of Equation (14) and the steady-state solution of Equation (55) can be added together to produce a general solution for $v_n(t)$:

$$v_n(t) = V_{nt}e^{-\gamma t}\sin(\omega_1 t + \varphi_t) + V_{ns}\cos(\omega t + \varphi_s) \qquad (69)$$

Where γ, ω_1, V_{ns} and φ_s are defined by Equations (60), (20), (65) and (63) respectively. The two remaining variables, V_{nt} and φ_t define the transient part of the solution and must be chosen to suit the initial conditions, i.e. the conditions at $t = t_f$, the instant at which the fault occurs. If the fault resistance is equal to or lower than the threshold value given by Equation (45), then the transient solution of Equation (52) should be substituted for Equation (14). The discussion below deals with fault resistance values higher than the threshold value.

Transient parameters to suit initial conditions

Two initial conditions can be used to define the transient response fully:

1. Zero neutral voltage: Assume the system was in steady state with no neutral voltage prior to the fault. Because of the presence of capacitance from each phase to earth, none of the network voltages can change instantaneously when the fault occurs, so the neutral voltage is also zero at the instant of the fault, i.e. $v_n(t_f) = 0$. This condition can be used with Equation (69) to provide a relationship between V_{nt} and φ_t.

$$0 = V_{nt}e^{-\gamma t_f}\sin(\omega_1 t_f + \varphi_t) + V_{ns}\cos(\omega t_f + \varphi_s) \qquad (70)$$

2. Zero neutral inductor current: The current in inductor L_n cannot change instantaneously. Given the neutral voltage was zero at all instants prior to the fault occurrence, the current in the inductor is also zero when the fault occurs.
   Equation (25) shows that since $v_n(t = t_f) = 0$ and $i_n(t = t_f) = 0$, then $\frac{dv_n(t)}{dt} = 0$ at $t = t_f$.

   To use the second of these initial conditions, we use Equations (15) and (56):

$$0 = V_{nt}e^{-\gamma t_f}\omega_1 \cos(\omega_1 t_f + \varphi_t) - V_{nt}e^{-\gamma t_f}\gamma \sin(\omega_1 t_f + \varphi_t) - \omega V_{ns} \sin(\omega t_f + \varphi_s) \tag{71}$$

Equations (70) and (71) can be solved together to find V_{nt} and φ_s. Consider two cases:

Fault occurrence at zero phase voltage

In this case, the fault occurs at a zero crossing in the Phase A voltage waveform and $t_f = 0$. Equations (70) and (71) can then be written:

$$0 = V_{nt} \sin(\varphi_t) + V_{ns} \cos(\varphi_s) \tag{72}$$

$$0 = V_{nt}\omega_1 \cos(\varphi_t) - V_{nt}\gamma \sin(\varphi_t) - \omega V_{ns} \sin(\varphi_s) \tag{73}$$

If the network is perfectly tuned, then by Equation (63), $\varphi_s = {}^{\pi}/_{2}$. Using this in Equation (72) indicates that $\varphi_t = 0$, so using Equation (66), Equation (73) becomes:

$$V_{nt} = \frac{\omega}{\omega_1}\frac{R_{eq}}{R_f}V_p \tag{74}$$

Substituting these values for V_{nt} and φ_s into Equation (69), the full solution is thus:

$$v_n(t) = -\frac{R_{eq}}{R_f}V_p\left[\sin(\omega t) - \frac{\omega}{\omega_1}e^{-\gamma t}\sin(\omega_1 t)\right] \tag{75}$$

For high resistance faults, we know from Figure 7, $\omega \cong \omega_1$ so this result is a simple one:

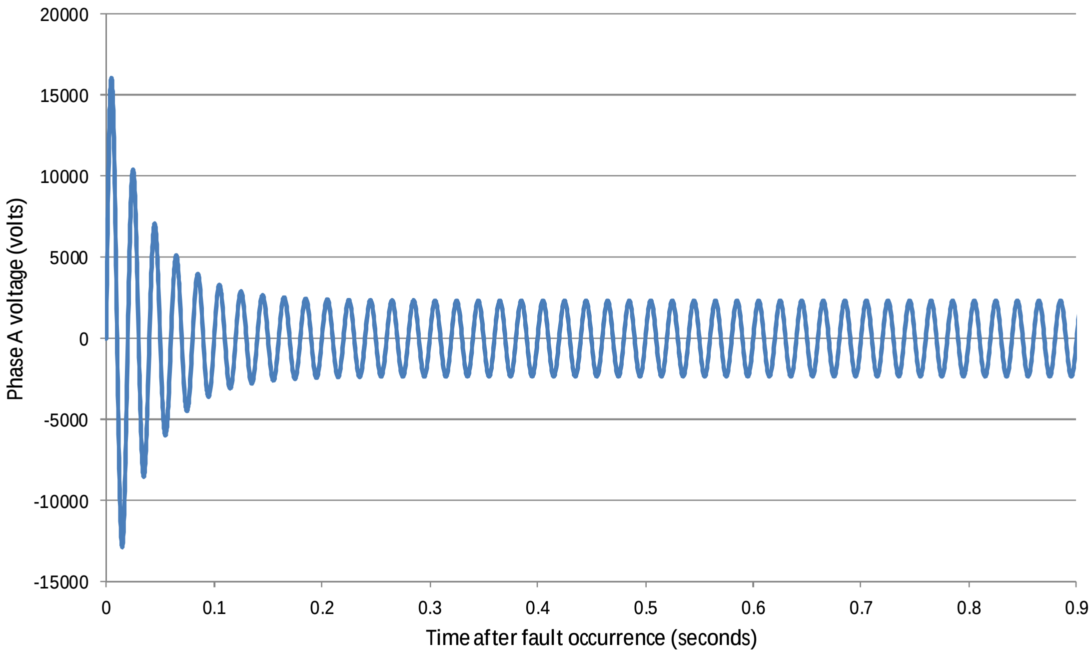
$$v_n(t) = -\frac{R_{eq}}{R_f}V_p(1 - e^{-\gamma t})\sin(\omega t) \tag{76}$$

Using Equation (1), the voltage on the faulted phase conductor is:

$$v_a(t) = V_p\left[1 - \frac{R_n}{R_f + R_n}(1 - e^{-\gamma t})\right]\sin(\omega t) \tag{77}$$

For lower resistance faults, both the neutral voltage and the phase voltage will depart slightly from this solution due to the slight difference between ω_1 and ω.

Figure 11: calculated test network faulted-phase voltage response to 600 ohm fault



Fault occurrence at peak phase voltage

In this case, the fault occurs at the peak of the Phase A voltage waveform, at $\omega t_f = \frac{\pi}{2}$ or $t_f = \frac{\pi}{2\omega}$. Equations (70) and (71) then become:

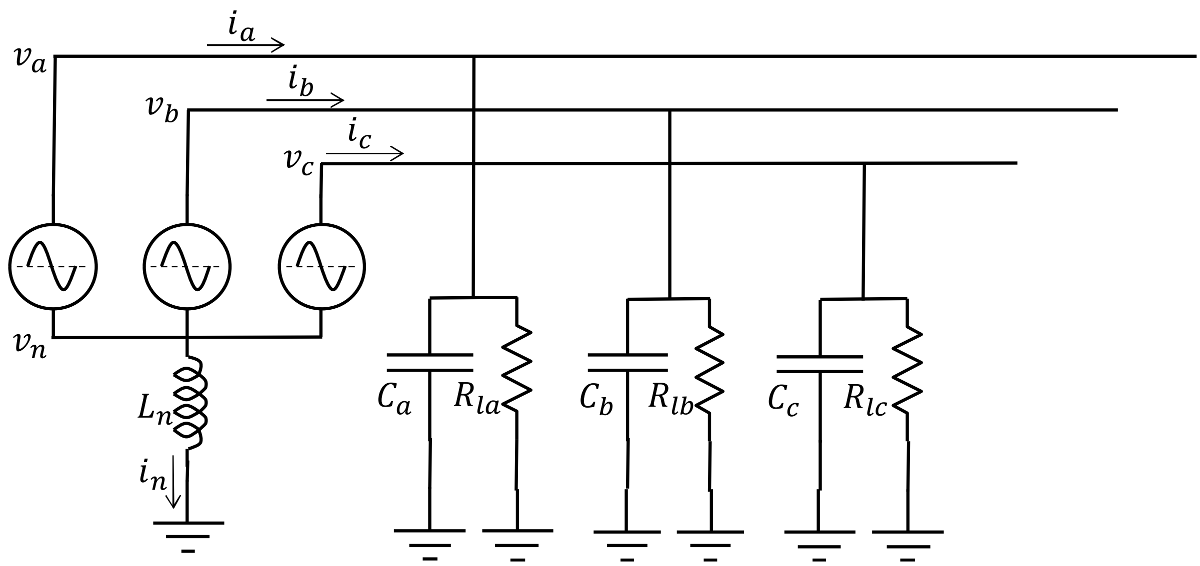
$$0 = V_{nt}e^{-\gamma\frac{\pi}{2\omega}} sin\left(\frac{\omega_1}{\omega}\frac{\pi}{2} + \varphi_t\right) + V_{ns}\, cos\left(\frac{\pi}{2} + \varphi_s\right) \tag{78}$$

$$0 = V_{nt}e^{-\gamma\frac{\pi}{2\omega}}\omega_1\, cos\left(\frac{\omega_1}{\omega}\frac{\pi}{2} + \varphi_t\right) - V_{nt}e^{-\gamma\frac{\pi}{2\omega}}\gamma\, sin\left(\frac{\omega_1}{\omega}\frac{\pi}{2} + \varphi_t\right) - \omega V_{ns}\, sin\left(\omega\frac{\pi}{2\omega} + \varphi_s\right) \tag{79}$$

The derivation of the solution is somewhat lengthy and is not shown here. When compared to the 'fault at zero crossing' solution, it has the same decay time constant and the same slight beating effect, with a 5ms time-shift of the transient with respect to the steady-state solution.

Effect of network imbalance

If the capacitance to earth and leakage to earth of the three phases is not balanced and the source voltages are unbalanced, some additional complexity enters the analysis. The network now looks like this:



The currents that flow in the three phase conductors are:

$$i_a(t) = \frac{v_n(t)}{R_{la}} + C_a \frac{dv_n(t)}{dt} + \frac{V_{pa}}{R_{la}} \sin(\omega t) + V_{pa}\omega C_a \cos(\omega t) \qquad (80)$$

$$i_b(t) = \frac{v_n(t)}{R_{lb}} + C_b \frac{dv_n(t)}{dt} + \frac{V_{bp}}{R_{lb}} \sin\left(\omega t + \frac{2\pi}{3}\right) + V_{pb}\omega C_b \cos\left(\omega t + \frac{2\pi}{3}\right) \qquad (81)$$

$$i_c(t) = \frac{v_n(t)}{R_{lc}} + C_c \frac{dv_n(t)}{dt} + \frac{V_{pc}}{R_{lc}} \sin\left(\omega t + \frac{4\pi}{3}\right) + V_{pc}\omega C_c \cos\left(\omega t + \frac{4\pi}{3}\right) \qquad (82)$$

The three phase sets of sinusoidal terms no longer cancel to zero. The effect of this is to add a sinusoidal current term to the equation. This acts as a forcing function to produce a standing neutral displacement voltage. This additional current term due to imbalance can be summarised by:

$$i_{imb}(t) = I_{imb} \sin(\omega t + \theta) = \frac{V_{pa}}{R_{la}} \sin(\omega t) + \frac{V_{pb}}{R_{lb}} \sin\left(\omega t + \frac{2\pi}{3}\right) + \frac{V_{pc}}{R_{lc}} \sin\left(\omega t + \frac{4\pi}{3}\right) + V_{pa}\omega C_a \cos(\omega t) + V_{pb}\omega C_b \cos\left(\omega t + \frac{2\pi}{3}\right) + V_{pc}\omega C_c \cos\left(\omega t + \frac{4\pi}{3}\right) \qquad (83)$$

So the equation for $v_n(t)$ becomes:

$$i_{imb}(t) = \frac{1}{L_n} \int v_n(t)\ dt + \frac{1}{R_n} v_n(t) + C_n \frac{dv_n(t)}{dt} \qquad (84)$$

Where total network capacitance and total network resistance are now defined as:

Total network damping resistance: $R_n = \frac{1}{\left[\frac{1}{R_{la}} + \frac{1}{R_{lb}} + \frac{1}{R_{lc}}\right]}$ (85)

Total nework capacitance: $C_n = C_a + C_b + C_c$ (86)

Comparing equation (84) to equation (28) reveals that the imbalance current $i_{imb}(t)$ acts as a forcing function in a similar way to the forcing function created by fault current in equation (28). As it is made up of 50Hz sinusoidal terms, it will produce a sinusoidal neutral voltage. The phase and magnitude of this forcing function will depend on the degree of imbalance and its precise distribution across the three phases.

If the neutral inductor is perfectly tuned to resonate with total network capacitance at 50Hz, then the standing neutral voltage displacement will be:

$$v_n(t) = -R_n I_{imb} \sin(\omega t + \theta) \tag{87}$$

Any neutral voltage displacement due to a fault will add vectorially to this standing value to produce the final neutral voltage displacement in the faulted state. This means the standing neutral voltage displacement will influence the residual fault current, so it is of some interest to calculate it.

Trigonometric identities can be used to derive the components of $i_{imb}(t)$ that are in phase with the Phase A voltage $v_a(t)$ (those that contain $\sin \omega t$) and those that are in quadrature with it (those that contain $\cos \omega t$). The applicable identities are set out in the following table:

$\sin(A + B) = \sin A \cos B + \cos A \sin B$	$\sin\left(\omega t + \frac{2\pi}{3}\right) = \sin \omega t \cos \frac{2\pi}{3} + \cos \omega t \sin \frac{2\pi}{3}$ $\sin\left(\omega t + \frac{4\pi}{3}\right) = \sin \omega t \cos \frac{4\pi}{3} + \cos \omega t \sin \frac{4\pi}{3}$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$	$\cos\left(\omega t + \frac{2\pi}{3}\right) = \cos \omega t \cos \frac{2\pi}{3} - \sin \omega t \sin \frac{2\pi}{3}$ $\cos\left(\omega t + \frac{4\pi}{3}\right) = \cos \omega t \cos \frac{4\pi}{3} - \sin \omega t \sin \frac{4\pi}{3}$
$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ and $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$	$\cos \frac{2\pi}{3} = -\frac{1}{2}$ and $\cos \frac{4\pi}{3} = -\frac{1}{2}$
$\sin\left(\omega t + \frac{2\pi}{3}\right) = -\frac{1}{2} \sin \omega t + \frac{\sqrt{3}}{2} \cos \omega t$ $\sin\left(\omega t + \frac{4\pi}{3}\right) = -\frac{1}{2} \sin \omega t - \frac{\sqrt{3}}{2} \cos \omega t$	$\cos\left(\omega t + \frac{2\pi}{3}\right) = -\frac{1}{2} \cos \omega t - \frac{\sqrt{3}}{2} \sin \omega t$ $\cos\left(\omega t + \frac{4\pi}{3}\right) = -\frac{1}{2} \cos \omega t + \frac{\sqrt{3}}{2} \sin \omega t$

These identities can be substituted into Equation (83) to transform the imbalance forcing function into a more useful form:

$$\begin{gathered} i_{imb}(t) = \left[\frac{V_{pa}}{R_{la}} - \frac{1}{2}\frac{V_{pb}}{R_{lb}} - \frac{1}{2}\frac{V_{pc}}{R_{lc}} - \frac{\sqrt{3}}{2} V_{pb}\omega C_b + \frac{\sqrt{3}}{2} V_{pc}\omega C_c\right] \sin(\omega t) + \\ \left[V_{pa}\omega C_a - \frac{1}{2} V_{pb}\omega C_b - \frac{1}{2} V_{pc}\omega C_c + \frac{\sqrt{3}}{2}\frac{V_{pb}}{R_{lb}} - \frac{\sqrt{3}}{2}\frac{V_{pc}}{R_{lc}}\right] \cos(\omega t) \end{gathered} \tag{88}$$

Equation (88) reveals that standing neutral displacement in an unbalanced network is generally zero only when network resistances, capacitances and voltages are all balanced. The industry focus is generally on capacitive imbalance due to two-wire spur lines. If we assume voltage and resistive imbalance is zero, the imbalance current due to capacitive imbalance is:

$$i_{imb}(t) = \frac{V_p \omega}{2} \left\{\sqrt{3}[C_c - C_b] \sin(\omega t) + [2C_a - C_b - C_c] \cos(\omega t)\right\} \tag{89}$$

The magnitude of this current is:

$$|i_{imb}(t)| = \frac{V_p \omega}{2} \sqrt{\{3[C_c - C_b]^2 + [2C_a - C_b - C_c]^2\}}$$

$$|i_{imb}(t)| = V_p \omega \sqrt{\left\{{C_c}^2 + {C_b}^2 + {C_a}^2 + C_b(C_c - C_a) + C_c(C_b - C_a)\right\}}$$

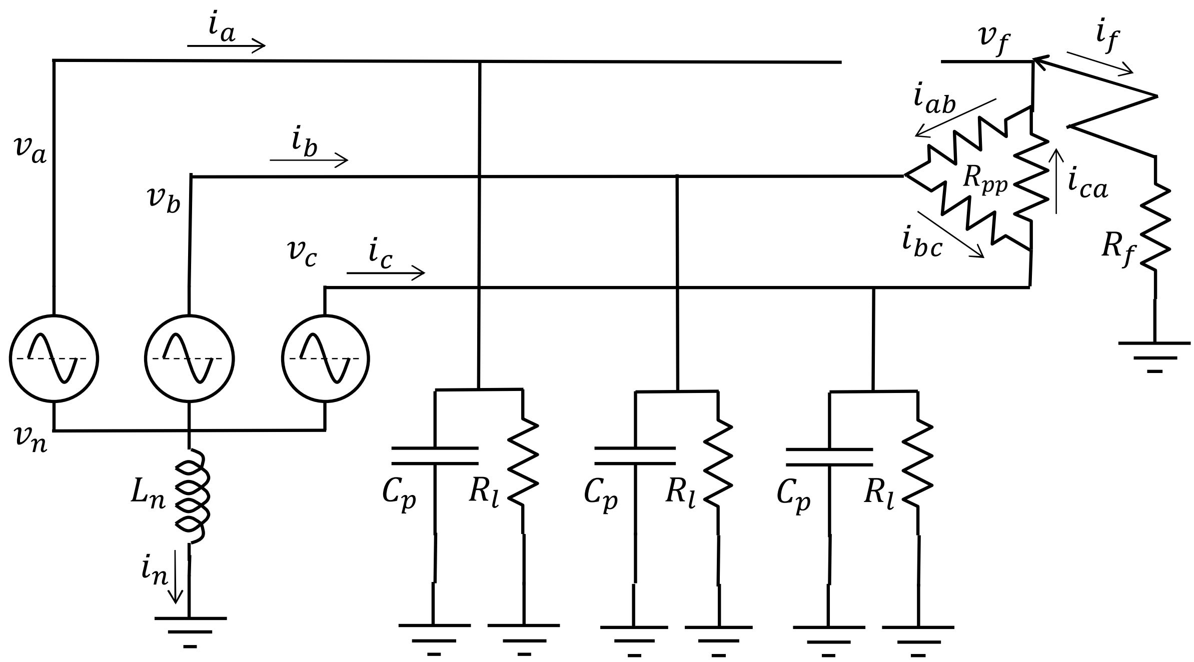
In the test network the standing neutral voltage was often around 900 volts, implying (using the values in Table 1), a standing neutral current of two amps if the network was solidly earthed.

Analysis of 'wire down' back-fed faults

When a conductor breaks and falls to the ground, if the fallen conductor is connected to the network at the 'downstream' pole of the fallen span and there is no earth fault on the 'upstream' side of the break, then earth fault current can flow due to the customer load connected between phases downstream of the break.

This is called a back-fed 'wire down' earth fault and is illustrated in Figure 12.

Figure 12: Wire-down back-fed earth fault



The equations that apply in this situation are:

Voltage equations

Equations (1) to (3) apply, as well as an equation for the voltage on the fallen conductor:

$$v_a(t) = v_n(t) + V_p\,sin(\omega t) \quad (90)$$

$$v_b(t) = v_n(t) + V_p\,sin\left(\omega t + \frac{2\pi}{3}\right) \quad (91)$$

$$v_c(t) = v_n(t) + V_p\,sin\left(\omega t + \frac{4\pi}{3}\right) \quad (92)$$

$$v_f(t) = R_f\,i_f(t) \quad (93)$$

Current equations

The current equations now have additional terms and additional equations for load currents and fault current:

$$i_a(t) = \frac{v_n(t)}{R_l} + C_p\frac{dv_n(t)}{dt} + \frac{V_p}{R_l}sin(\omega t) + V_p\omega C_p\,cos(\omega t) \quad (94)$$

$$i_b(t) = \frac{v_n(t)}{R_l} + C_p\frac{dv_n(t)}{dt} + \frac{V_p}{R_l}sin\left(\omega t + \frac{2\pi}{3}\right) + V_p\omega C_p\,cos\left(\omega t + \frac{2\pi}{3}\right) + i_{bc}(t) - i_{ab}(t) \quad (95)$$

$$i_c(t) = \frac{v_n(t)}{R_l} + C_p\frac{dv_n(t)}{dt} + \frac{V_p}{R_l}sin\left(\omega t + \frac{4\pi}{3}\right) + V_p\omega C_p\,cos\left(\omega t + \frac{4\pi}{3}\right) + i_{ca}(t) - i_{bc}(t) \quad (96)$$

$$i_n(t) = \frac{1}{L_n}\int v_n(t)\;dt \quad (97)$$

$$i_f(t) = i_{ca}(t) - i_{ab}(t) \quad (98)$$

$$i_{ab}(t) = \frac{1}{R_{pp}}[v_f(t) - v_b(t)] \quad (99)$$

$$i_{bc}(t) = \frac{1}{R_{pp}}[v_b(t) - v_c(t)] \quad (100)$$

$$i_{ca}(t) = \frac{1}{R_{pp}}[v_c(t) - v_f(t)] \quad (101)$$

Applying Kirchhoff's current law to the neutral connection gives:

$$0 = i_n(t) + i_a(t) + i_b(t) + i_c(t) \quad (102)$$

This is:

$$0 = \frac{1}{L_n}\int v_n(t)\;dt + \frac{1}{R_n}v_n(t) + C_n\frac{dv_n(t)}{dt} + i_{ca}(t) - i_{ab}(t) \quad (103)$$

The last term in Equation (103) is simply $i_f(t)$ which equals:

$$i_f(t) = i_{ca}(t) - i_{ab}(t) = \frac{1}{R_{pp}}\left[v_b(t) + v_c(t) - 2v_f(t)\right] = \frac{1}{R_{pp}}[v_b(t) + v_c(t)] - \frac{2R_f}{R_{pp}} i_f(t) \quad (104)$$

Equation (104) yields:

$$i_f(t) = \frac{1}{R_{pp}+2R_f}[v_b(t) + v_c(t)] \quad (105)$$

Using Equations (91) and (92) for $[v_b(t) + v_c(t)]$ gives:

$$i_f(t) = \frac{1}{R_{pp}+2R_f}\left[2v_n(t) + V_p\left\{sin\left(\omega t + \frac{2\pi}{3}\right) + sin\left(\omega t + \frac{4\pi}{3}\right)\right\}\right] \quad (106)$$

To simplify this, we can use: $0 = \sin(\omega t) + \sin\left(\omega t + \frac{2\pi}{3}\right) + \sin\left(\omega t + \frac{4\pi}{3}\right)$, to get:

$$i_f(t) = \frac{1}{R_{pp}+2R_f}\left[2v_n(t) - V_p\, sin(\omega t)\right] \quad (107)$$

The system equation is now:

$$\frac{1}{(R_{pp}+2R_f)} V_p\, sin(\omega t) = \frac{1}{L_n}\int v_n(t)\, dt + \left[\frac{1}{R_n} + \frac{1}{\left(\frac{1}{2}R_{pp}+R_f\right)}\right] v_n(t) + C_n \frac{dv_n(t)}{dt} \quad (108)$$

After differentiation and division by C_n, this gives:

$$\frac{\omega V_p}{2R_{fbf}C_n} cos(\omega t) = \frac{1}{L_nC_n} v_n(t) + \frac{1}{R_{eqbf}C_n}\frac{dv_n(t)}{dt} + \frac{d^2v_n(t)}{dt^2} \quad (109)$$

Where:

$$R_{eqbf} = \frac{1}{\frac{1}{R_n}+\frac{1}{R_{fbf}}} = \frac{R_nR_{fbf}}{R_n+R_{fbf}} \quad (110)$$

$$R_{fbf} = \frac{1}{2}R_{pp} + R_f \quad (111)$$

Comparing Equation (108) to Equation (25) and comparing Equation (109) to Equation (26) indicates the system response will be of the same form as that of a system with a normal earth fault, with some adjustments to the parameters:

1. The effective fault resistance is the actual fault resistance R_f plus half the phase-to-phase resistance R_{pp} that represents the load on the network downstream of the fault location.
2. The forcing function has half the amplitude of that which applies with a normal fault on Phase A and is in quadrature to it, i.e. the neutral voltage displacement will be about half what would occur if the fault was not back-fed. This means fault detection of back-fed faults is at best only half as sensitive as detection of normal faults of the same fault resistance.

If the network is perfectly tuned to the system frequency, then at resonance, the capacitive and inductive current terms in Equation (108) cancel each other, leaving:

$$v_n(t) = \frac{R_{eqbf}}{2R_{fbf}} V_p \sin(\omega t) \quad (112)$$

Which allows the fault current to be determined using Equation (107):

$$i_f(t) = -\frac{V_p \sin(\omega t)}{(2R_n + R_{pp} + 2R_f)} \quad (113)$$

Detection of back-fed earth faults

The challenge of back-fed fault detection is well known. The following analysis compares the capability of resonant earthed networks that use neutral voltage rise for earth fault detection with solidly earthed networks that use zero-sequence over-current detection.

Resonant earthed networks

In a resonant earthed network, the fault is detected by neutral voltage displacement. Let the threshold of detection be k%. Using Equation (112), detection will occur if:

$$\frac{R_{eqbf}}{2R_{fbf}} \geq \frac{k}{100} \quad (114)$$

The threshold can be drawn as a boundary on a map of nominal fault current (the fault current that would be drawn by the same 'wire on ground' fault as a normal, i.e. not back-fed, fault of the same fault resistance in a solidly earthed network) versus customer load downstream of the fault. The two dimensions in Figure 13 are:

Y-coordinate: Nominal fault current equal to 12.7kV divided by the fault resistance $\mathrm{R_f}$.

X-coordinate: Downstream load equal to $\frac{3 \times 22^2}{\mathrm{R_{pp}}}$, where $\mathrm{R_{pp}}$ is the phase-phase load resistance.

Figure 13: Detection of back-fed faults in resonant earthed networks (test network parameters)

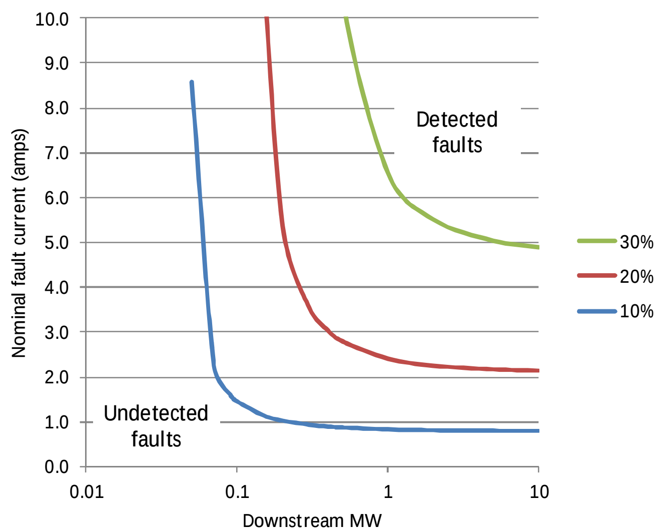


Figure 13 demonstrates that:

- On the test network with standard fault detection sensitivity settings (k=30%) a resonant earthed system will only detect a back-fed five amp fault if downstream load is greater than about five megawatts. This might only occur if the fault is close to the zone substation and customer demand is high.
- If higher sensitivity is used (k=20%) then a five amp back-fed fault can be detected even if downstream customer load is as low as 200 kilowatts.
- At maximum feasible sensitivity (k=10% - not generally feasible with today's products) a resonant earthed system will detect a one amp back-fed fault if customer load downstream of the fault is greater than about 200kW or a five amp back-fed fault with downstream customer demand as low as 75kW.

Comparison: Solidly earthed networks

With no inductor and the neutral point solidly connected to earth, $v_n(t) = 0$ and the voltage and current equations are:

$$v_a(t) = V_p\, sin(\omega t) \tag{115}$$

$$v_b(t) = V_p\, sin\left(\omega t + \frac{2\pi}{3}\right) \tag{116}$$

$$v_c(t) = V_p\, sin\left(\omega t + \frac{4\pi}{3}\right) \tag{117}$$

$$v_f(t) = R_f\, i_f(t) \tag{118}$$

$$i_a(t) = \frac{V_p}{R_l} sin(\omega t) + V_p \omega C_p\, cos(\omega t) \tag{119}$$

$$i_b(t) = \frac{V_p}{R_l} sin\left(\omega t + \frac{2\pi}{3}\right) + V_p \omega C_p\, cos\left(\omega t + \frac{2\pi}{3}\right) + i_{bc}(t) - i_{ab}(t) \tag{120}$$

$$i_c(t) = \frac{V_p}{R_l} sin\left(\omega t + \frac{4\pi}{3}\right) + V_p \omega C_p\, cos\left(\omega t + \frac{4\pi}{3}\right) + i_{ca}(t) - i_{bc}(t) \tag{121}$$

$$i_n(t) = -[i_a(t) + i_b(t) + i_c(t)] = [i_{ab}(t) - i_{ca}(t)] = -i_f(t) \tag{122}$$

$$i_{ab}(t) = \frac{1}{R_{pp}}[v_f(t) - v_b(t)] \tag{123}$$

$$i_{ca}(t) = \frac{1}{R_{pp}}[v_c(t) - v_f(t)] \tag{124}$$

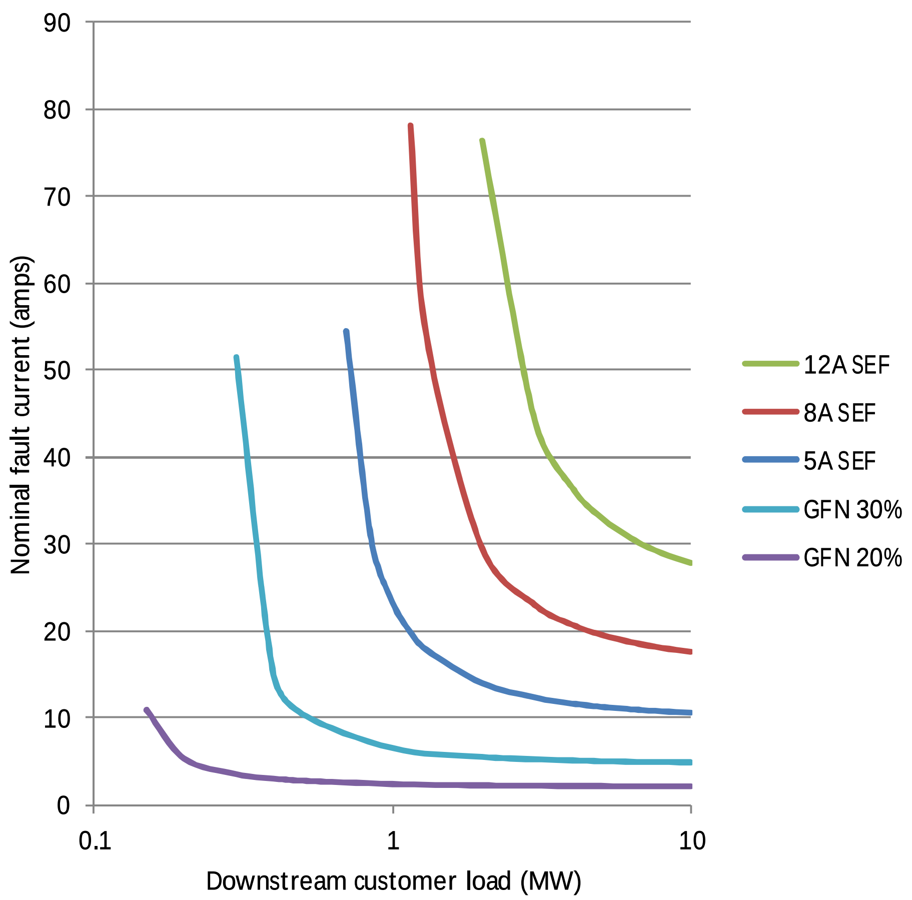
$$i_f(t) = i_{ca}(t) - i_{ab}(t) = \frac{1}{R_{pp}}[v_b(t) + v_c(t) - 2v_f(t)] \tag{125}$$

So:

$$i_n(t) = \frac{V_p}{(R_{pp}+2R_f)} sin(\omega t) \qquad (126)$$

In a similar manner to Figure 13 above, the Sensitive Earth Fault (SEF) protection system detection threshold can be mapped for various settings of SEF relay minimum operation current. Figure 14 shows the fault detection boundary for various SEF minimum operating current settings compared with normal resonant earth setting (30%) and heightened sensitivity setting (20%).

Figure 14: comparison: detection of back-fed faults in solidly earthed versus resonant earthed networks



This comparison indicates that, compared to solidly earthed networks with SEF protection, resonant earthed networks have superior capability to detect back-fed faults in challenging situations where:

- Downstream customer load is low; and/or,
- Fault resistance is high.

Detection of the fault is only the first step. Most suppliers of resonant earthing equipment recommend the feeder be tripped as soon as a back-fed fault is detected, i.e. residual current compensation is not used even if available.

Comparison of theory and time domain simulation

An example of each earth fault type (based on the test network parameters listed in Table 1) was tested using time domain simulation in Circuit Lab Pro. For this test, it was assumed:

- Total customer load downstream of the broken conductor is 0.5MW, i.e. 167kW per phase.
- Fault resistance is 2,000 ohms, i.e. a high impedance fault (nominally 6.4 amps)
- Network damping resistance is 4,000 ohms, i.e. earth leakage from each phase is 12,000 ohms.

The phase-to-phase resistance due to customer load is:

$$R_{pp} = \frac{(22{,}000\ volts)^2}{166{,}666\ watts} = 2{,}904\ ohms \qquad (127)$$

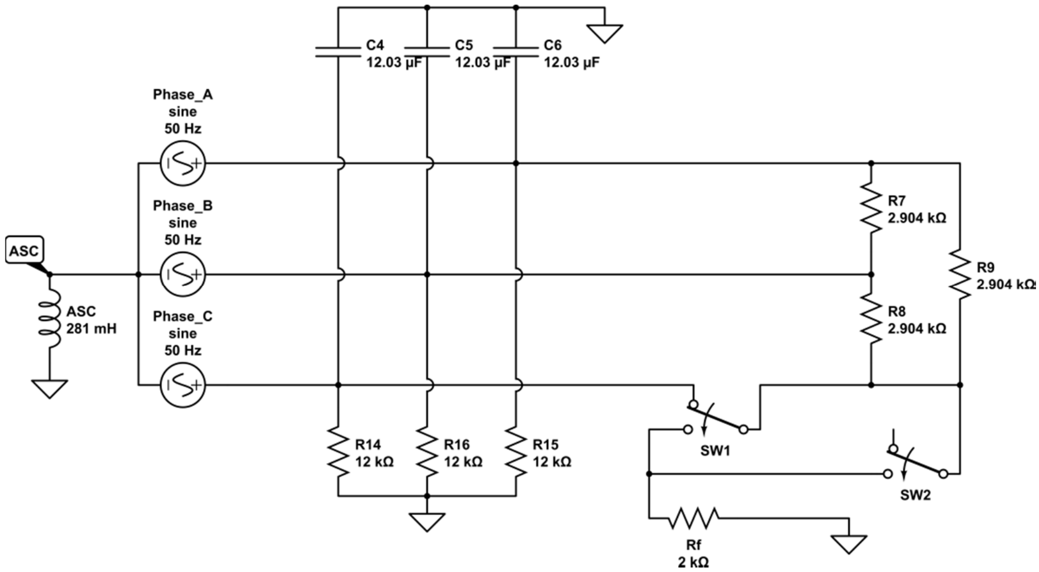
So for the back-fed fault, the new values of fault resistance and damping resistance are:

$$R_{fbf} = 2{,}000 + \frac{1}{2} \times 2{,}904 = 3{,}452\ ohms \qquad (128)$$

$$R_{eqbf} = \frac{3{,}452 \times 4{,}000}{(3{,}452+4{,}000)} = 1{,}853\ ohms \qquad (129)$$

The time domain simulation model is shown in Figure 15.

Figure 15: time domain simulation model for earth faults in resonant earthed network



The model was run for one second of simulated time with a time step of one or two microseconds. The two switches allowed both normal and back-fed earth faults to be simulated. Time constants were measured from plots as the time it took for the amplitude of the 50Hz waveform to reach 63% of its final value, i.e. this measurement is not as precise as other values derived from the simulations.

Back-fed earth fault comparison

The comparison results for a back-fed fault are shown in Table 7. In this simulation, switch SW2 remains open and switch SW1 changes state 40 milliseconds into the simulation.

Table 7: comparison of earth fault theory and simulation results: back-fed fault

Parameter	Equation	Theory result	Simulation result	Difference (%)
Fault current	(113)	0.852 A (rms)	0.862 A (rms)	1.2 %
Neutral voltage	(112)	3.409 kV (rms)	3.375 kV (rms)	1.0 %
Phase C voltage	(90)	16.112 kV (rms)	16.01 kV (rms)	0.6 %
Time constant	$\tau = 2R_{eqbf}C_n$	0.1338 seconds	0.1385 seconds (est)	3.7 %

It can be seen that the action of the resonant earthing is to reduce the voltage on the two undamaged phases and in consequence, increase the voltage on the phase with the fallen conductor.

Normal earth fault comparison

The comparison results for a normal earth fault are shown in Table 8. In this simulation, switch SW1 remains in the state shown in Figure 13 and switch SW2 changes state 40 milliseconds into the simulation to apply the fault.

Table 8: comparison of earth fault theory and simulation results: normal earth fault

Parameter	Equation	Theory result	Simulation result	Difference (%)
Fault current	(41)	2.117 A (rms)	2.140 A (rms)	1.1 %
Neutral voltage	(29)	8.467 kV (rms)	8.421 kV (rms)	0.5 %
Phase C voltage	(30)	4.234 kV (rms)	4.309 kV (rms)	1.8 %
Time constant	$\tau = 2R_{eq}C_n$	0.108 seconds	0.097 seconds	11.3 %

Comparison of theory and distributed element simulation model

A generic rural network was defined. It comprised five identical 22kV feeders of 93 kilometre length. Each feeder started in a medium size country town, extended to supply a second smaller town 30 kilometres distant using typical ‘backbone’ aluminium conductor, then extended a further 25 kilometres using thinner aluminium conductor to supply an even smaller town, then extended 30 kilometres into sparsely populated territory using steel conductor. The model of each feeder comprised 19 sections of 5 kilometres each.

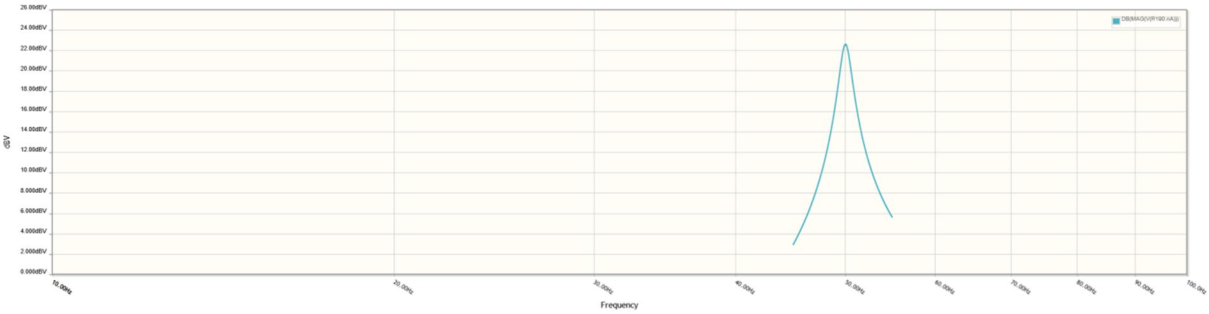
Customer load and mutual capacitance between phases was not represented in the model. The leakage resistance to earth from overhead line sections was assumed to be of magnitude equal to 2% of the capacitive current to earth of the same section.

The model grouped identical sets of elements to produce the structure shown in Figure 17 on page 36.

Tuning and DC shunt resistance

The ASC tuning was checked using a frequency domain sweep and it was confirmed that a coil inductance of 665mH created resonance in the model at 50.08 Hertz as shown in Figure 16. This gave a value for C_n of 15.187µF which is 0.6% larger than the calculated total of all capacitances to earth in the model, 15.095µF.

Figure 16: generic rural network model – coil tuning check



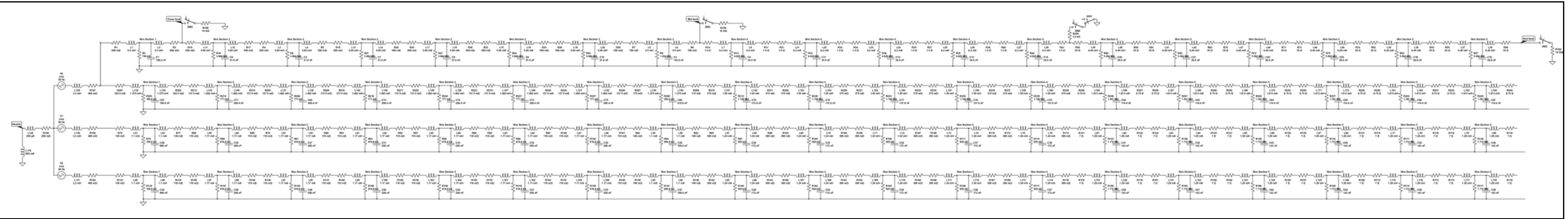
A neutral DC injection simulation confirmed the effective total shunt resistance to earth is 10,542 ohms. However, the time-domain simulations demonstrated the value of R_n was much lower than this due to energy losses in the series resistance of the phase conductors.

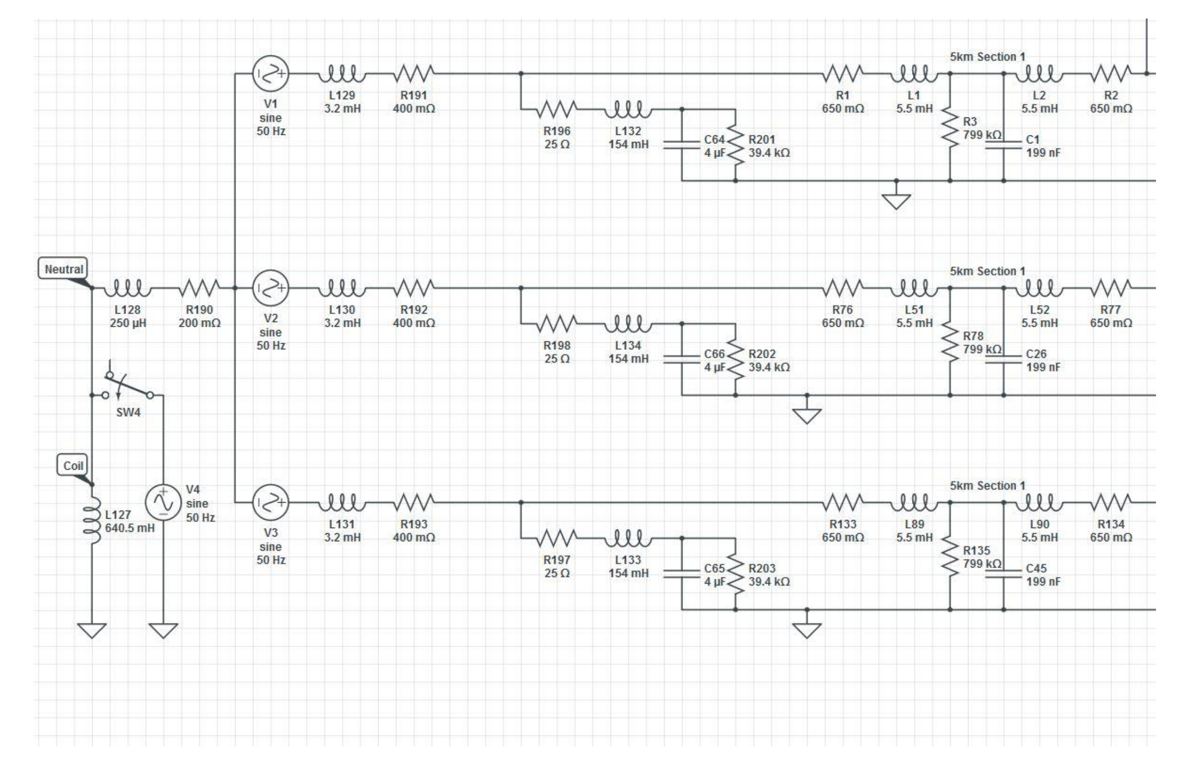
Simulation of a remote 6400 ohm earth fault

An initial simulation was for 1.0 seconds with a step length of 5µs. A fault of resistance 6400 ohms was applied at the junction of the aluminium and steel conductor, i.e. 65 kilometres from the source zone substation. The waveforms of neutral voltage and fault current are shown in Figure 18.

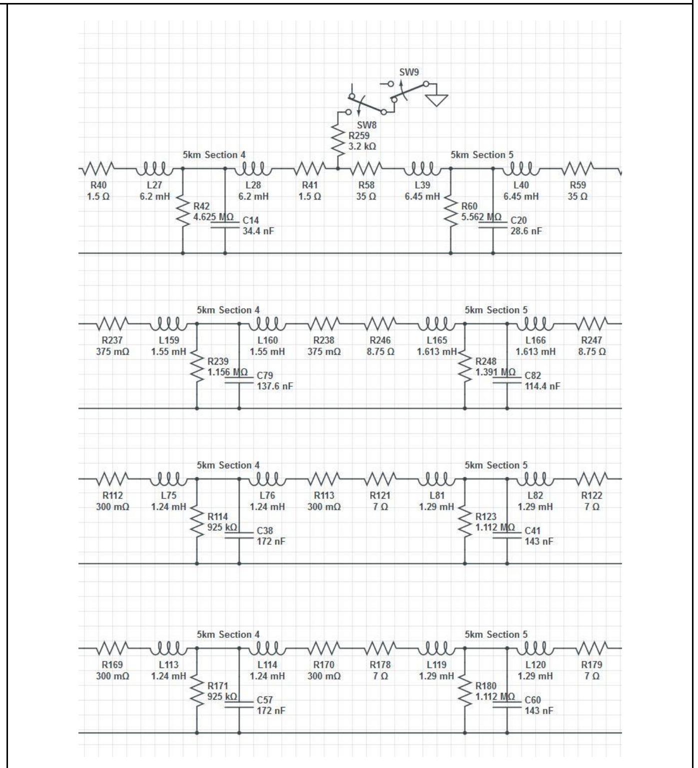
The final value of neutral displacement voltage was 7.066 kilovolts and the final value of fault current was 0.8779 amps. The final value of the voltage on the faulted phase (at the fault location) was 5.674kV. All these measurements are rms quantities calculated by Circuit Lab Pro over a 100.0ms interval towards the end of the simulation period. These values were used to obtain a value for the network damping resistance and compare the calculated transient decay time constant against theory.

Figure 17: model of generic rural network



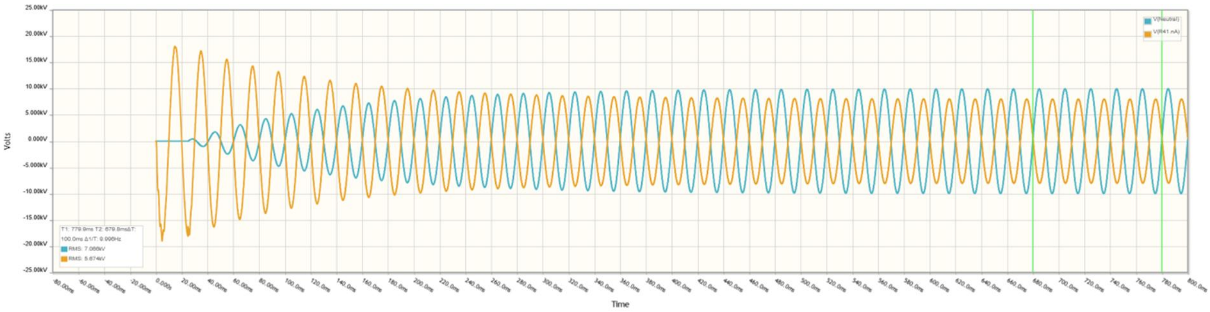


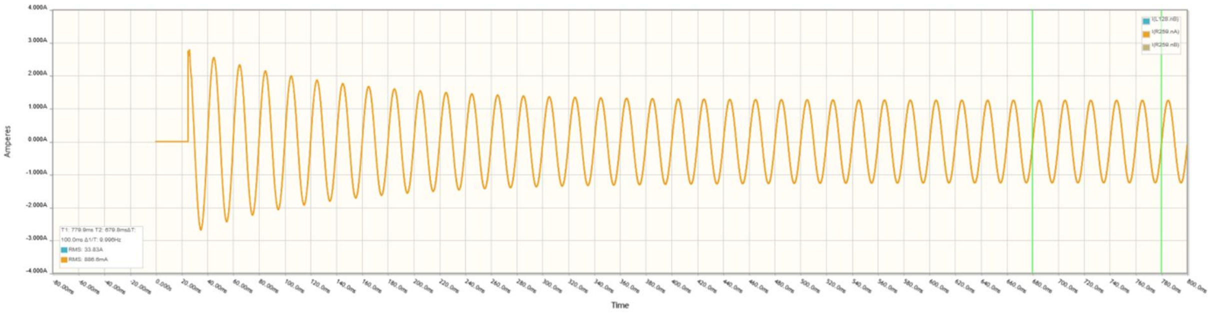
Model detail: source zone substation



Model detail: fault location

Figure 18: 6400 ohm fault at remote location on generic network (5μs step)





The simulation results were analysed using the theory developed for a lumped element network representation to see if the two are consistent. The results of this analysis are shown in Table 9:

Table 9: analysis of simulation results for 6400 ohm remote fault on distributed element model

Quantity	Derivation	Result	Comment
R_n	*Faulted phase volts using Equation (30)*	*7,927 ohms*	*The three values for R_n fall within a 1.8% range, with an average value of 8,007 ohms. This value is 24% less than the effective total DC shunt resistance of the network to earth.*
R_n	*Neutral volts using Equation (29)*	*8,024 ohms*	
R_n	*Fault current using Equation (39)*	*8,069 ohms*	
R_{eq}	*R_n (average)*	*3,557 ohms*	
τ_{on}	*R_n and C_n using Equations (19), (27)*	*108.0 ms*	*This is 0.2% different to the 107.8 ms value measured from the neutral voltage waveform.*

The analysis uses the following re-written equations:

$$R_n = R_f \left[\frac{1}{\frac{V_p}{|v_n(t)|} - 1} \right] \tag{130}$$

$$R_n = R_f \left[\frac{V_p}{|v_a(t)|} - 1 \right] \tag{131}$$

$$R_n = \frac{V_p}{|i_f(t)|} - R_f \tag{132}$$

$$R_{eq} = \frac{R_n R_f}{(R_n + R_f)} \tag{133}$$

$$\tau_{on} = 2R_{eq}C_n \tag{134}$$

$$\tau_{off} = 2R_nC_n \tag{135}$$

The simulation demonstrates that:

- The generic rural network is quite sensitive - a 0.9 amp fault located 65 km from the substation produces a neutral displacement voltage equal to 56% of nominal. This because of the network's small 'electrical size' due to the relatively minor proportion of underground cable.
- The value of network damping resistance derived from the simulation is significantly lower than the DC shunt resistance to earth. This is because it has to account for energy losses in series resistance in the network conductors.

Effect of simulation time step length

The neutral voltage, faulted-phase voltage and fault current were simulated over a longer period to allow analysis of the transient following removal of the fault. It proved not possible to do such long simulations at a time step of 5µs, so a 20µs time step was used to reduce computation times to reasonable levels. This changed the rms values of the quantities in the simulation somewhat as shown in Table 10.

Table 10: comparison of simulation results for different time step durations

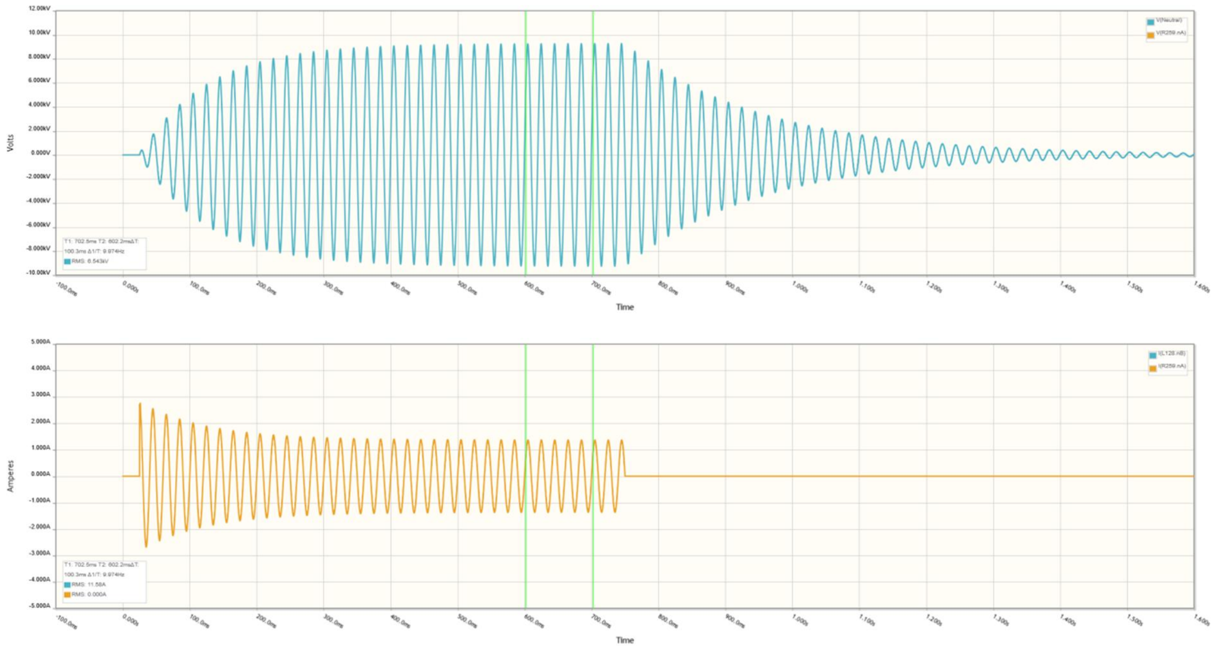
Time step:	5 µs	20 µs	Change
Neutral displacement voltage:	7.066 kV	6.549 kV	-7.3%
Faulted phase voltage:	5.674 kV	6.190 kV	+9.1%
Fault current:	0.8779 A	0.9672 A	+10.2%

So a longer step length in the simulation increases the fault current and reduces the neutral displacement, i.e. it reduces the value of the network damping resistance. The fault current implies a value for R_n of 6,733 ohms, which is 17% less than the value derived from the fault current in the simulation with a 5µs step.

Transient response to fault removal

The waveforms for a 6,400 ohm earth fault occurrence at 0.025 seconds and removal at 0.750 seconds are shown in Figure 19. These are from the 2.5 second simulation with a step size of 20µs.

Figure 19: generic rural network - occurrence and removal of 6400 ohm fault



The transient time constants were measured by inspecting the waveforms to find the time at which the transient had reduced by 63.2% of its initial magnitude. The precise time was derived by interpolation between the two nearest waveform peaks. The measured time constants and the theoretical calculated values derived from the values in Table 10 using the average value of R_n derived from the measured values set out in the 20µs column of Table 10.

Table 11: fault occurrence and removal time constants (6400 ohm fault 20µs time step)

Quantity	Derivation	Result	Comment
R_n	*Faulted phase volts using Equation (30)*	*6,733 ohms*	
R_n	*Neutral volts using Equation (29)*	*6,812 ohms*	*Values are within a 1.2% range, average value is 6,759 ohms. This is 36% less than the total DC shunt resistance of the network to earth.*
R_n	*Fault current using Equation (39)*	*6,733 ohms*	
R_{eq}	*R_n (average) and R_f*	*3,287 ohms*	
τ_{on}	*R_{eq} and C_n using Equations (19), (27)*	*99.8 ms*	*This is 1.2% shorter than the 101 ms value measured from the neutral voltage waveform.*
τ_{off}	*R_n and C_n using Equation (19)*	*206.9 ms*	*This is 3.5% longer than the 200 ms value measured from the neutral voltage waveform.*

These comparisons show that while time domain simulation is moderately sensitive to step length, the measurement derived from it form an internally self-consistent set.

Simulation of remote 3,200 ohm earth fault

The same simulation was run for a 3,200 ohm earth fault. The fault current was 1.273 amps and the neutral displacement was 8.633kV. The analysis results are shown in Table 12.

Table 12: fault occurrence and removal time constants (3200 ohm remote fault 20us time step)

Quantity	Derivation	Result	Comment
R_n	Neutral volts using Equation (29)	6,789 ohms	The average R_n is 6,784 ohms and the two derived values are within a 0.3% range around this value
R_n	Fault current using Equation (39)	6,778 ohms	
R_{eq}	R_n (average) and R_f	2,174 ohms	
τ_{on}	R_{eq} and C_n using Equations (19), (27)	66.0 ms	This is 0.2% longer than the 65.9 ms value measured from the neutral voltage waveform.
τ_{off}	R_n and C_n using Equation (19)	206.1 ms	This is 1.3% longer than the 203 ms value measured from the neutral voltage waveform.

So the agreement of the simulation with theory is again excellent.

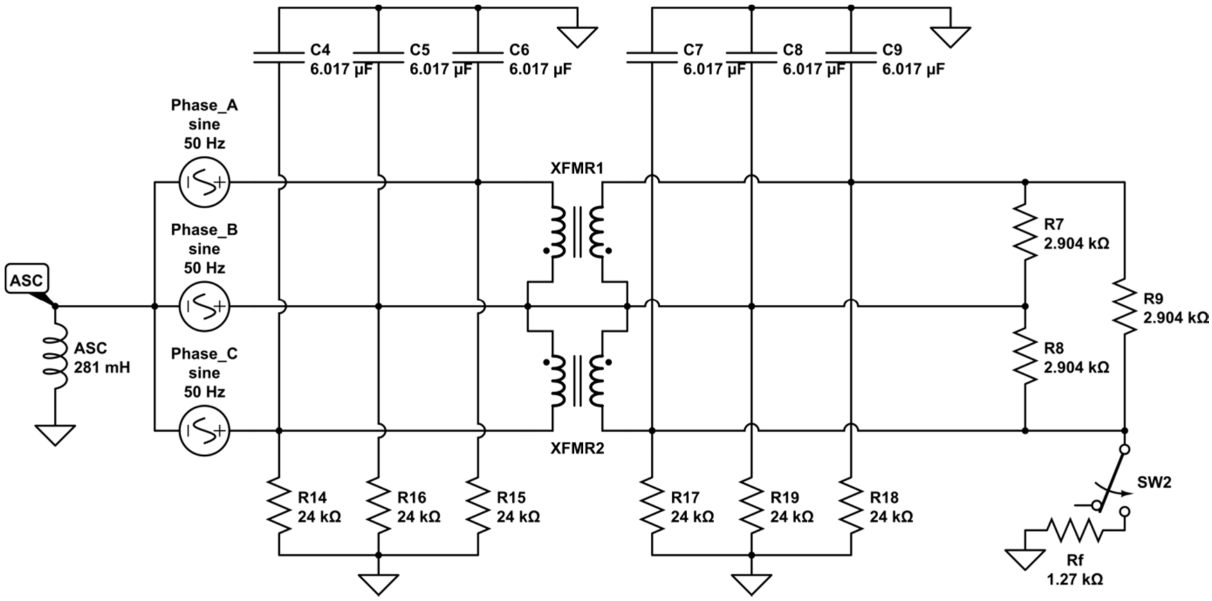
Conclusion: simulations support theory

Overall, it was concluded that the theory developed from the lumped element model is useful in deriving results from simulations using a much more complex distributed element model of a generic rural network. The assumptions used in the theoretical development appear sound.

Line capacitance beyond an open-delta voltage regulator

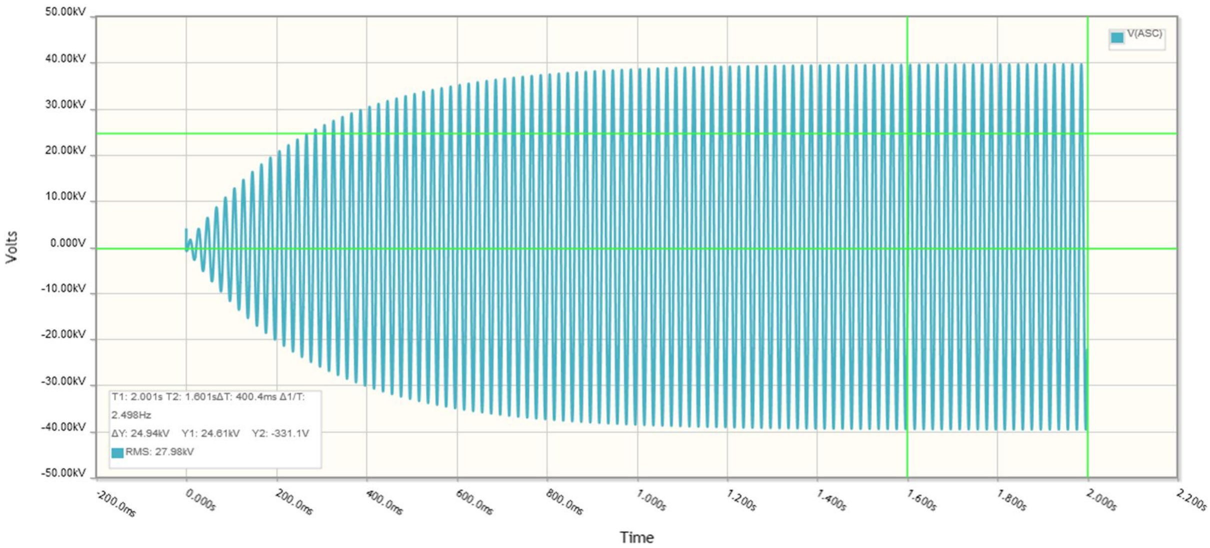
Normally voltage regulators are positioned mid-way along the length of long rural high voltage feeders so that part of the total network capacitance to earth is located downstream of them. This situation is illustrated by the time domain simulation model shown in Figure 20.

Figure 20: capacitance to earth beyond an open-delta voltage regulator



Without any fault on this system, i.e. switch SW2 open, the neutral voltage displacement can be extreme as shown in Figure 21 where it reaches 28.0 kV with a balanced turns ratio of 1.1/1.1 and downstream load of 0.5MW. The network time constant is 0.275 seconds. The neutral voltage rise is not dependent on downstream load – exactly the same result is obtained when the load is removed.

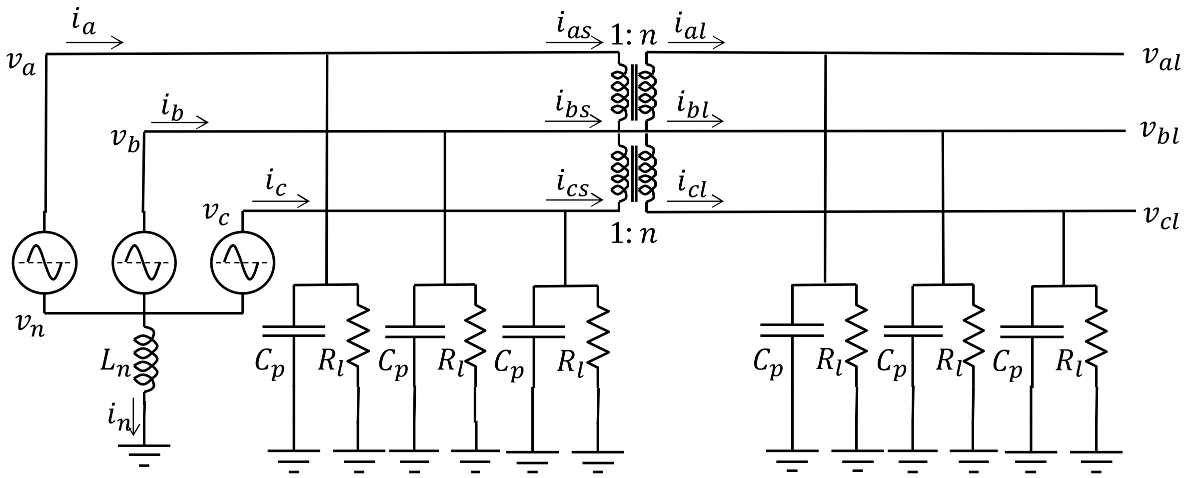
Figure 21: neutral voltage rise with capacitance to earth downstream of open-delta regulator



To understand the cause of such a large over-voltage, we can use the same analysis technique set out in earlier sections of this document. We ignore any downstream customer load and analyse the network in an un-faulted state. For simplicity, we assume half the network-to-earth capacitance is downstream of the open-delta regulator and half upstream.

For theoretical analysis, we use the model illustrated in Figure 22.

Figure 22: open-delta regulator with both upstream and downstream line capacitance and leakage



To avoid complexity in the equations, the same phase-to-earth capacitance and leakage resistance has been assumed on either side of the regulator. It should be recognised that the parameters shown must be halved if total network capacitance and leakage is to be similar to previous analyses. Because of the non-unity turns ratio of the voltage regulator, there are more equations required to fully describe the model.

Voltage equations

The equations for voltages shown in Figure 22 are:

Upstream:

$$v_a(t) = v_n(t) + V_p\, sin(\omega t) \tag{136}$$

$$v_b(t) = v_n(t) + V_p\, sin\left(\omega t + \frac{2\pi}{3}\right) \tag{137}$$

$$v_c(t) = v_n(t) + V_p\, sin\left(\omega t + \frac{4\pi}{3}\right) \tag{138}$$

$$v_a(t) + v_b(t) + v_c(t) = 3v_n(t) \tag{139}$$

$$v_a(t) + v_c(t) = 3v_n(t) - v_b(t) \tag{140}$$

Downstream:

$$v_{al}(t) = v_{bl}(t) + n[v_a(t) - v_b(t)] = nv_a(t) - (n-1)v_b(t) \tag{141}$$

$$v_{bl}(t) = v_b(t) \tag{142}$$

$$v_{cl}(t) = v_{bl}(t) + n[v_c(t) - v_b(t)] = nv_c(t) - (n-1)v_b(t) \tag{143}$$

This gives a result that will prove useful in simplifying the analysis:

$$[v_{al}(t) + v_{cl}(t) + v_{bl}(t)] = 3v_n(t) - 3(n-1)V_p\, sin\left(\omega t + \frac{2\pi}{3}\right) \tag{144}$$

Current equations

The equations for currents shown in Figure 22 are:

Upstream:

$$i_a(t) = \frac{1}{R_l} v_a(t) + C_p \frac{dv_a(t)}{dt} + i_{as}(t) \tag{145}$$

$$i_b(t) = \frac{1}{R_l} v_b(t) + C_p \frac{dv_b(t)}{dt} + i_{bs}(t) \tag{146}$$

$$i_c(t) = \frac{1}{R_l} v_c(t) + C_p \frac{dv_c(t)}{dt} + i_{cs}(t) \tag{147}$$

Downstream:

$$i_{al}(t) = \frac{1}{R_l} v_{al}(t) + C_p \frac{dv_{al}(t)}{dt} \tag{148}$$

$$i_{bl}(t) = \frac{1}{R_l} v_{bl}(t) + C_p \frac{dv_{bl}(t)}{dt} \tag{149}$$

$$i_{cl}(t) = \frac{1}{R_l} v_{cl}(t) + C_p \frac{dv_{cl}(t)}{dt} \tag{150}$$

$$[i_{al}(t) + i_{al}(t) + i_{al}(t)] = \frac{1}{R_l}[v_{al}(t) + v_{cl}(t) + v_{bl}(t)] + C_p \frac{d}{dt}[v_{al}(t) + v_{cl}(t) + v_{bl}(t)] \tag{151}$$

Voltage regulator:

$$i_{as}(t) + i_{bs}(t) + i_{cs}(t) = i_{al}(t) + i_{bl}(t) + i_{cl}(t) \tag{152}$$

At supply transformer neutral:

$$i_n(t) = \frac{1}{L_n} \int v_n(t)\ dt \tag{153}$$

System equation

The system equation is an equation in $v_n(t)$ derived by applying Kirchhoff's Current Law to the neutral connection point:

$$0 = i_n(t) + i_a(t) + i_b(t) + i_c(t) \tag{154}$$

Using Equations (145) to (147) and Equations (136) to (138), this is:

$$0 = \frac{1}{L_n}\int v_n(t)\ dt + \frac{3}{R_l}v_n(t) + 3C_p\frac{dv_n(t)}{dt} + [i_{as}(t) + i_{bs}(t) + i_{cs}(t)] \qquad (155)$$

Using Equations (152) and (151) followed by Equation (144) and remembering the total network quantities are: $\mathrm{C_n} = 6\mathrm{C_p}$ and $\mathrm{R_n} = \frac{\mathrm{R_l}}{6}$, we can write this:

$$\begin{aligned} 0 = \frac{1}{L_n}\int v_n(t)\ dt + \frac{1}{R_n}v_n(t) + C_n\frac{dv_n(t)}{dt} \\ - \frac{(n-1)V_p}{2R_n}\left[sin\left(\omega t + \frac{2\pi}{3}\right) + \omega R_n C_n\, cos\left(\omega t + \frac{2\pi}{3}\right)\right] \end{aligned} \qquad (156)$$

Differentiating and dividing by $\mathrm{C_n}$ gives:

$$-\frac{V_p(n-1)}{2C_nR_n}\left[sin\left(\omega t + \frac{2\pi}{3}\right) + \omega R_l C_p\, cos\left(\omega t + \frac{2\pi}{3}\right)\right] = \frac{1}{C_nL_n}v_n(t) + \frac{1}{C_nR_n}\frac{dv_n(t)}{dt} + \frac{d^2v_n(t)}{dt^2} \qquad (157)$$

Comparing this system equation with Equation (43) and Equation (11) reveals the following features:

1. The resonant frequency and transient time constant are both the same as those of an un-faulted network without a voltage regulator.
2. Even without a fault on the network, if $\mathrm{n} \neq 1$, there can be a substantial forcing function to produce a large neutral voltage displacement.

The amplitude of the neutral displacement in an un-faulted network with an open-delta voltage regulator can be derived from Equation (156), remembering that provided the network is perfectly tuned, at resonance the integral and derivative terms cancel to zero. This gives:

$$v_n(t) = -\frac{(n-1)V_p}{2}\left[sin\left(\omega t + \frac{2\pi}{3}\right) + \omega_0 R_n C_n\, cos\left(\omega t + \frac{2\pi}{3}\right)\right] \qquad (158)$$

$$|v_n(t)| = -\frac{(n-1)V_p}{2}\sqrt{1 + (\omega_0 R_n C_n)^2} \qquad (159)$$

For n=1.1 and the test network parameters, this is 28.8kV rms. The time domain simulation is shown in Figure 21 and the neutral voltage settles to 28.0kV rms after a build-up with an estimated time constant of 0.27 seconds, i.e. the discrepancy in amplitude is around 3% and the discrepancy in time constant is 7%.

It may not be realistic to postulate a 50/50 split of the total network capacitance distributed to either side of the voltage regulator. A simulation with a 90/10 split (upstream/downstream) was also run for a balanced turns ratio of 1.1 and the result was 4.65kV rms neutral displacement, rising to that level with a time constant of 0.28 seconds. A 10/90 split produced a neutral displacement of 51.3kV rms.

Comparison: Open delta regulators in solidly (and NER) earthed networks

If the neutral is solidly earthed, the neutral voltage is constrained to be zero. Equation (156) is then:

$$i_n(t) = \frac{(n-1)V_p}{2R_n}\left[\sin\left(\omega t + \frac{2\pi}{3}\right) + \omega R_n C_n \cos\left(\omega t + \frac{2\pi}{3}\right)\right] \tag{160}$$

$$|i_n(t)| = \frac{(n-1)V_p}{2R_n}\sqrt{1 + (\omega_0 R_n C_n)^2} \tag{161}$$

This is the neutral voltage displacement that would occur in the network if it were resonant earthed as given by Equation (158), divided by the network damping resistance $\mathrm{R_n}$.

A time-domain simulation with the inductor replaced by a one ohm resistor and a 90/10 upstream/downstream split of the network with a turns ratio of 1.1 shows a neutral current of 1.20A rms which correlates well (agrees within 3%) with the network damping resistance of 4,000 ohms and the neutral displacement voltage that would occur if the network were resonant earthed of 4.65kV rms. With an eight ohm NER (ten ohms total earth resistance) this current reduces by only 0.8%.

This result confirms that open-delta regulators create significant standing neutral current in solidly earthed and NER-earthed networks.

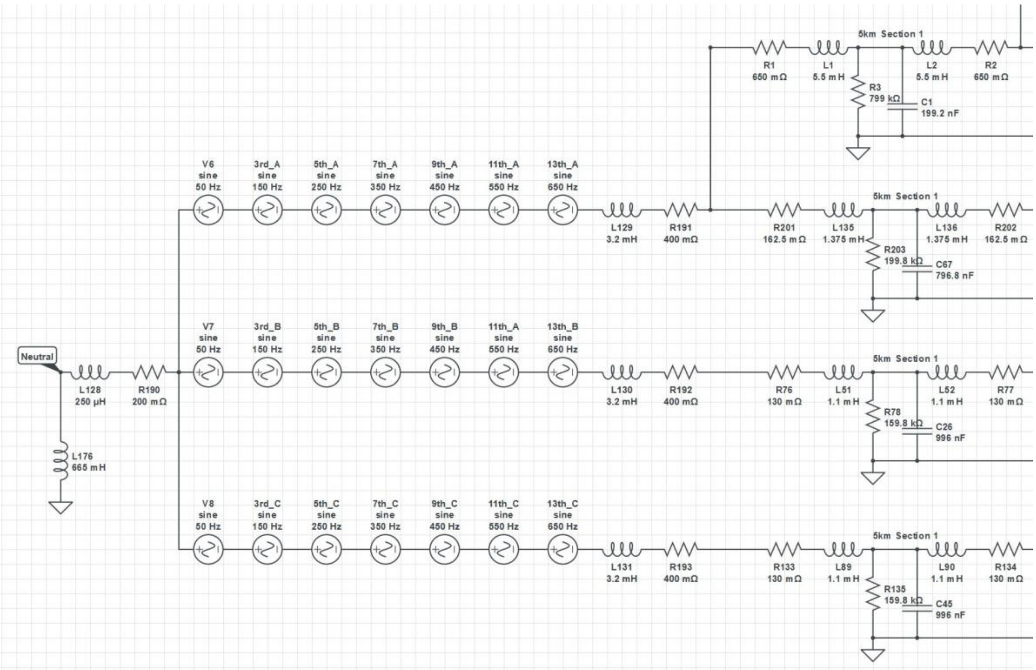
Effect of harmonics on fault current

The effect of network harmonics on earth fault current warrants consideration as harmonics may possibly increase the amount of energy released into the local environment at the fault location.

Simulation of network harmonics

The model shown in Figure 17 was modified to include sources of harmonics as shown in Figure 23.

Figure 23: model with harmonic distortion in network voltages



The amplitudes of the harmonic sources were set up as shown in Table 13.

Table 13: harmonic source settings

Harmonic	Amplitude (V_{pk})
Third	539
Fifth	323
Seventh	231
Ninth	180
Eleventh	147
Thirteenth	124

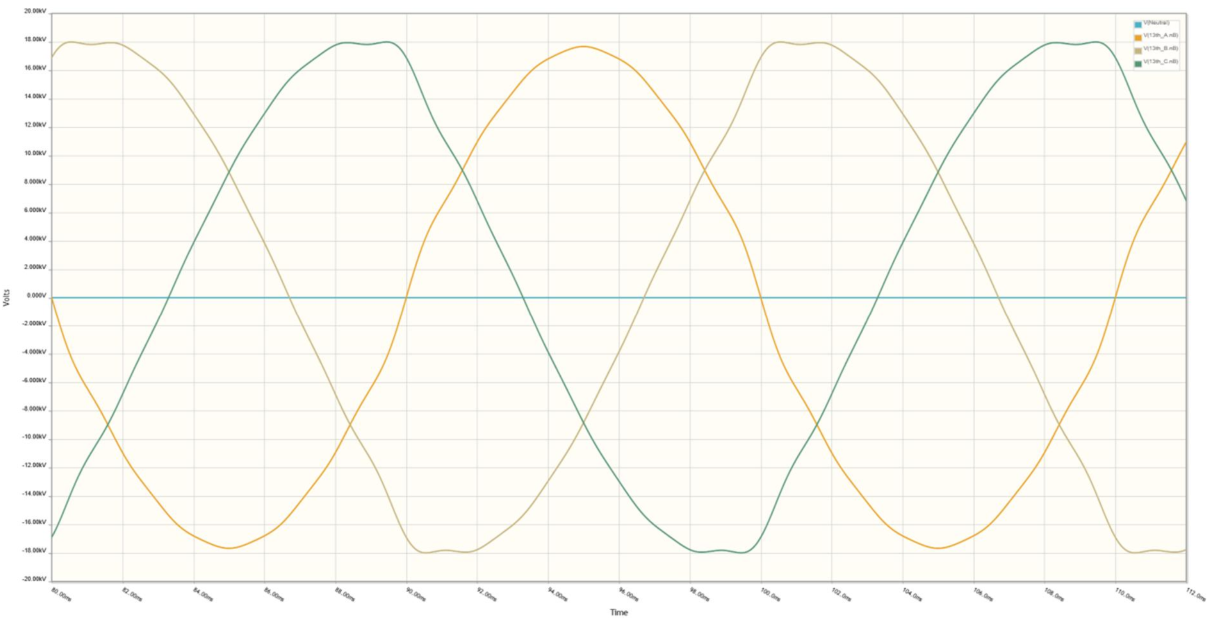
This harmonic profile is a classic ‘one over N’ in which the amplitude of each harmonic is inversely proportional to the harmonic number. The series starts with a third harmonic of amplitude equal to 3% of the fundamental amplitude. Total harmonic distortion in the phase voltages is 0.08%.

The phase of each harmonic was arbitrarily set to zero on Phase A, 120 degrees on Phase B and 240 degrees on Phase C. As the system contains no non-linear elements, the phase should be immaterial in the analysis, though variations in phase may dramatically change the visual appearance of the waveforms.

Confirmation of harmonic levels

A 200ms simulation of the un-faulted network was done with a time step of 5µs. The resulting three phase voltage waveforms are visibly distorted as can be seen in Figure 24.

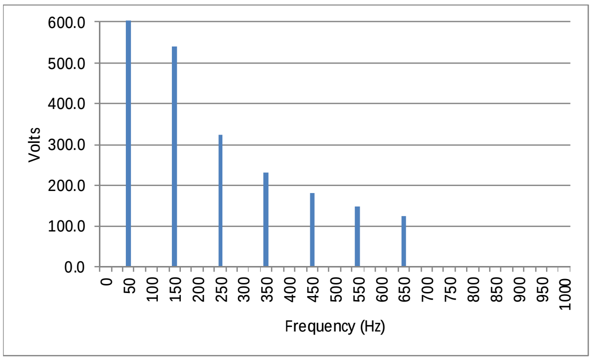
Figure 24: network voltages with harmonic distortion



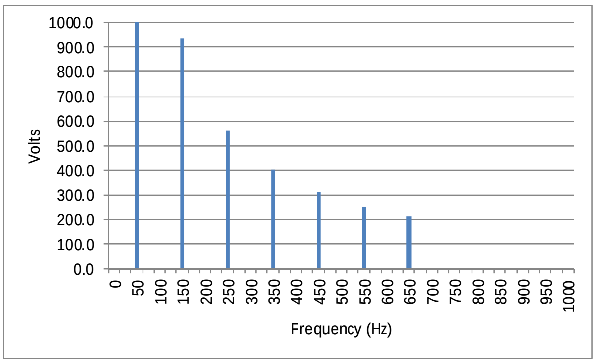
The spectra of the source phase and phase-to-phase voltages were calculated using a Fast Fourier

Transform algorithm. They were confirmed to be within one volt of the values set in the simulation model and listed in Table 13. Two examples are shown in Figure 25.

Figure 25: spectra of phase-to-neutral and phase-to-phase voltages



Phase A to neutral voltage spectrum



Phase A to Phase B voltage spectrum

Simulation of a high impedance earth fault on a network with harmonics

A remote (65km from the substation) 3,200 ohm earth fault was simulated for 500ms with a 5us time step. The resulting waveforms are shown in Figure 29 on the next two pages. The fault current and the phase-to-earth voltage at the fault location are rich in harmonics, while waveforms closer to the source substation are less so. The neutral voltage and coil current both have quite low harmonic levels as would be expected in a system with a strong resonance at 50Hz.

The fault current is 1.184 amps and the neutral displacement voltage is 9.057 kV. The waveforms and spectra for these two key quantities are shown here:

Figure 26: fault current waveform and spectrum – 3200 ohm remote earth fault

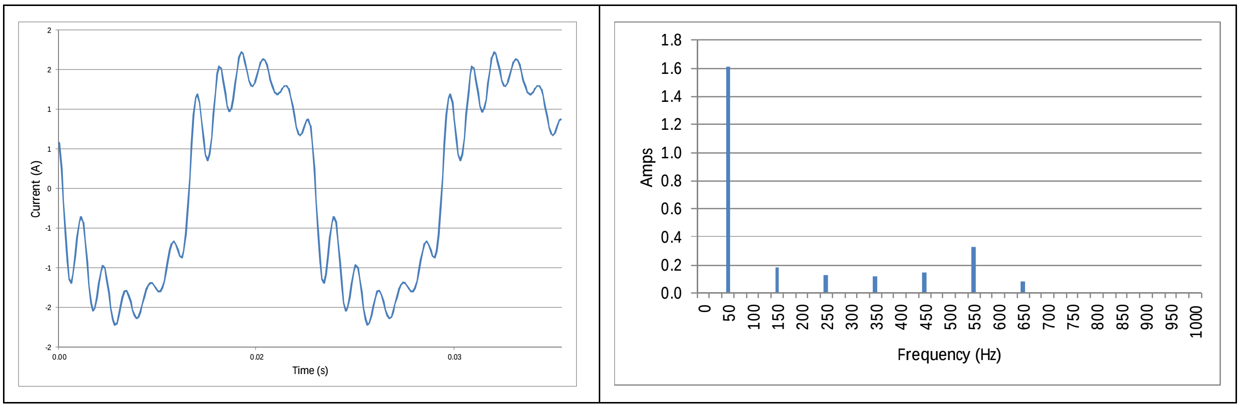
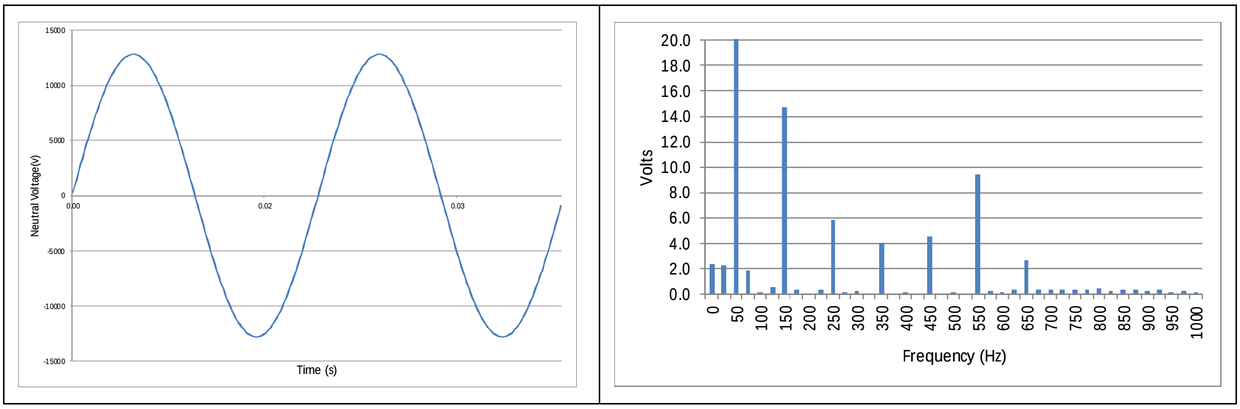


Figure 27: Neutral voltage waveform and spectrum – 3200 ohm remote earth fault



These simulation results show the following features:

- The ‘one on N’ source harmonics profile is only partially reflected in the network response.
- The 11th harmonic is greatly increased in amplitude relative to other harmonics.
- The level of harmonics in the neutral voltage is much lower than in the source phase voltages (maximum of 0.11% of the 50Hz value).
- The level of harmonics in the fault current is much higher than in the source phase voltages (from 11.3% to 20.1% of the 50Hz value). This is due to the suppression of the fundamental by the 50Hz ASC resonance.

The anomalous increase in the 11th harmonic is due to a 632Hz resonance in the network model, as shown in the network frequency response in Figure 28.

Figure 28: frequency response of network (for signal injection at fault location)

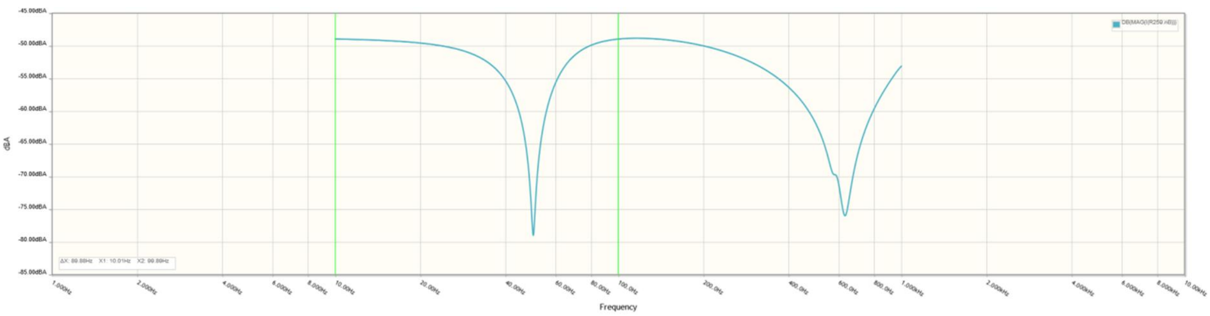
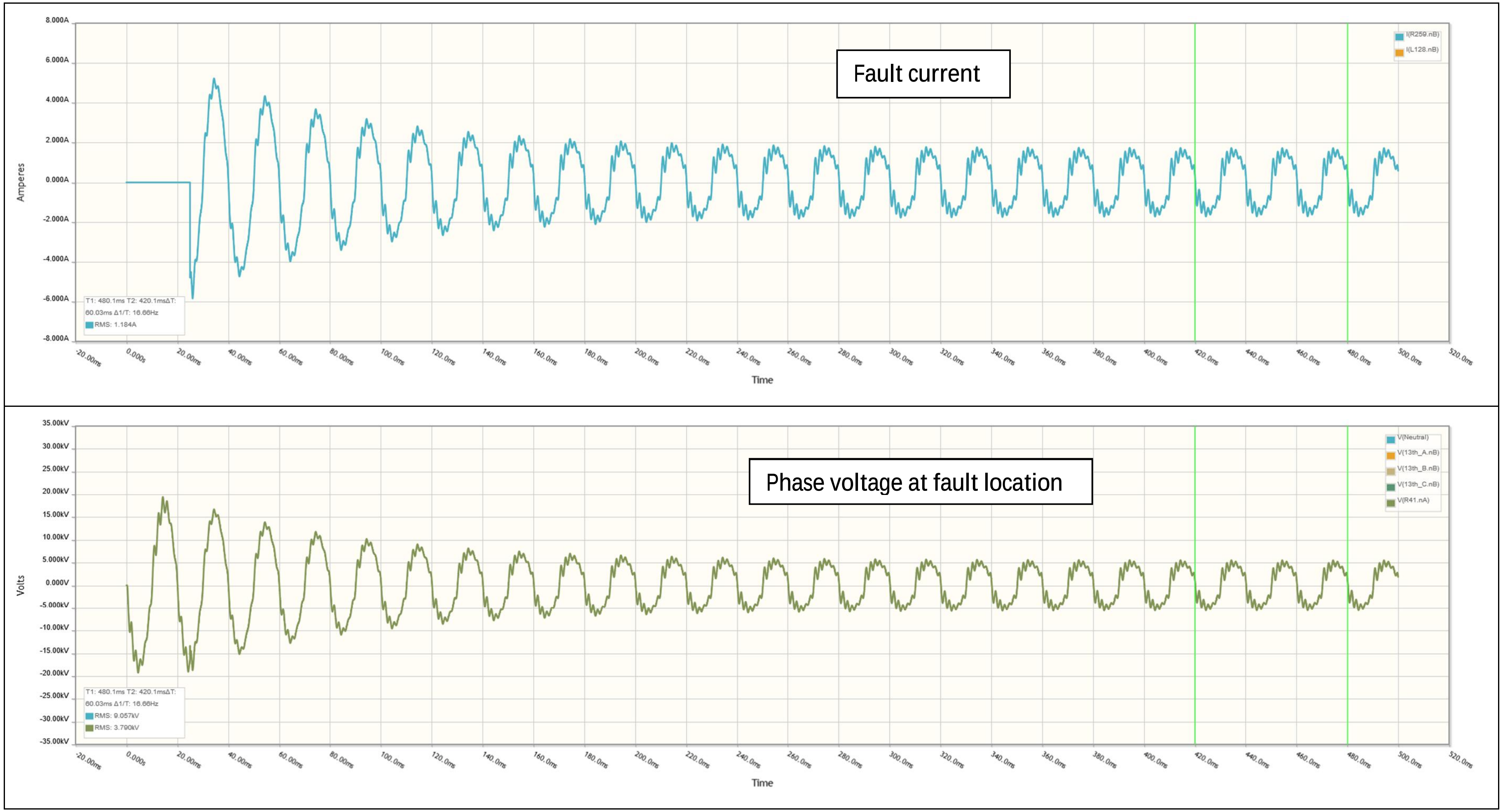
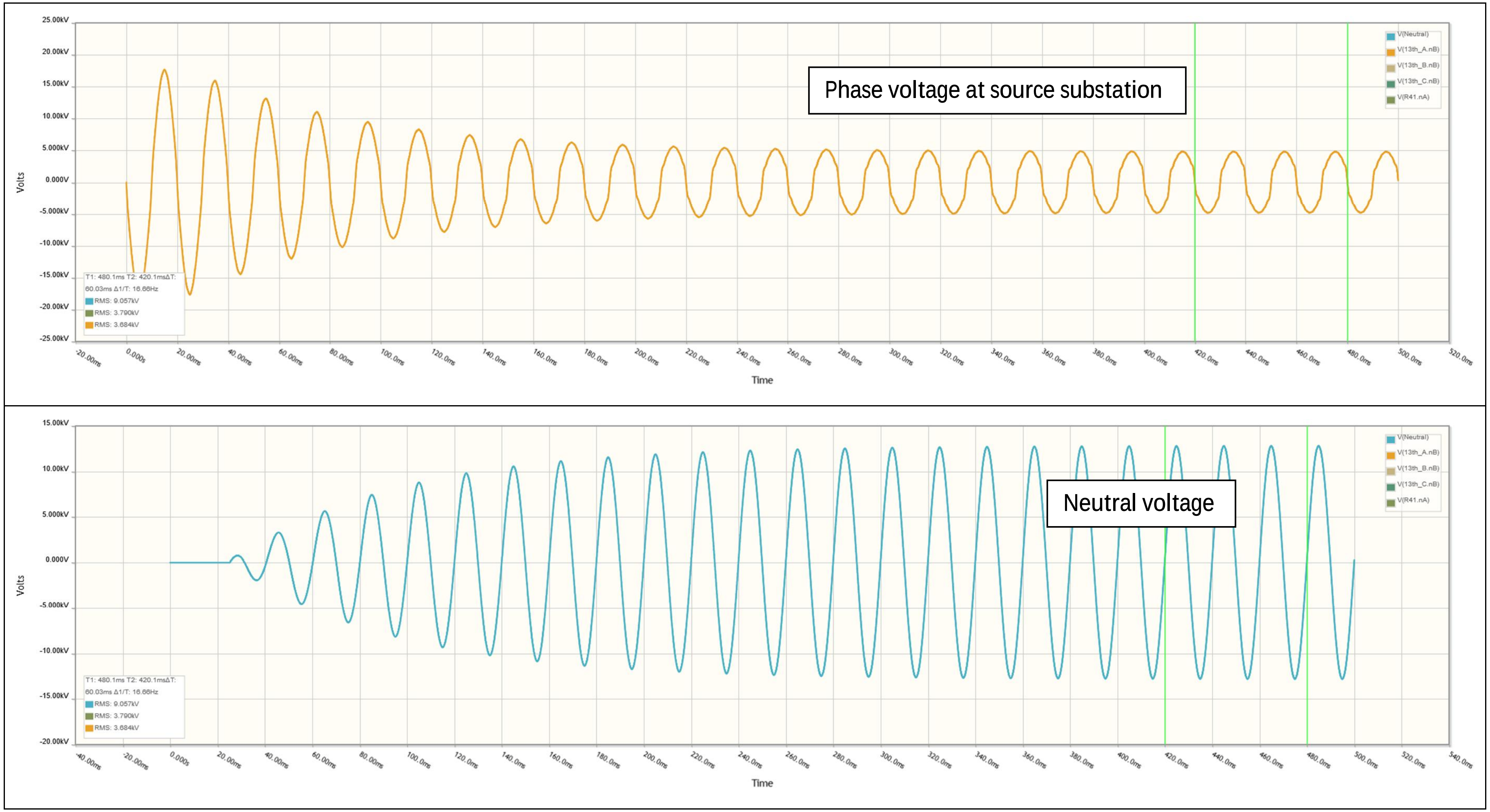


Figure 29: waveforms for a remote 3200 ohm fault in the presence of network voltage harmonics




Phase voltage at source substation
Volts
Time
V(Neutral)
V(13th_A.nB)
V(13th_B.nB)
V(13th_C.nB)
V(R41.nA)
T1: 480.1ms T2: 420.1msΔT: 60.03ms Δ1/T: 16.66Hz
RMS: 9.057kV
RMS: 3.790kV
RMS: 3.684kV
Neutral voltage

An identical simulation was performed with no harmonic sources. The fault current and neutral voltage with and without harmonics are shown in Table 14.

Table 14: Effect of harmonics on neutral voltage and fault current - remote 3200 ohm earth fault

Simulation	Neutral voltage	Fault current
With harmonics	9.057kV	1.184A
Without harmonics	9.060kV	1.141A
Difference	0.03%	3.8%

Because the neutral voltage is created by the 50Hz resonance of the ASC coil with the network, it is not surprising that harmonics have little effect. On the other hand, the harmonic currents generated by the fault are significant as the ASC does nothing to compensate for frequencies other than 50Hz.

Effect of harmonics on a low impedance fault

The simulation with the same harmonic profile was performed for a 250 ohm fault in the same location (65km from the substation). A simulation using identical parameters with no harmonics present provided a reference case.

Table 15: Effect of harmonics on neutral voltage and fault current - remote 250 ohm earth fault

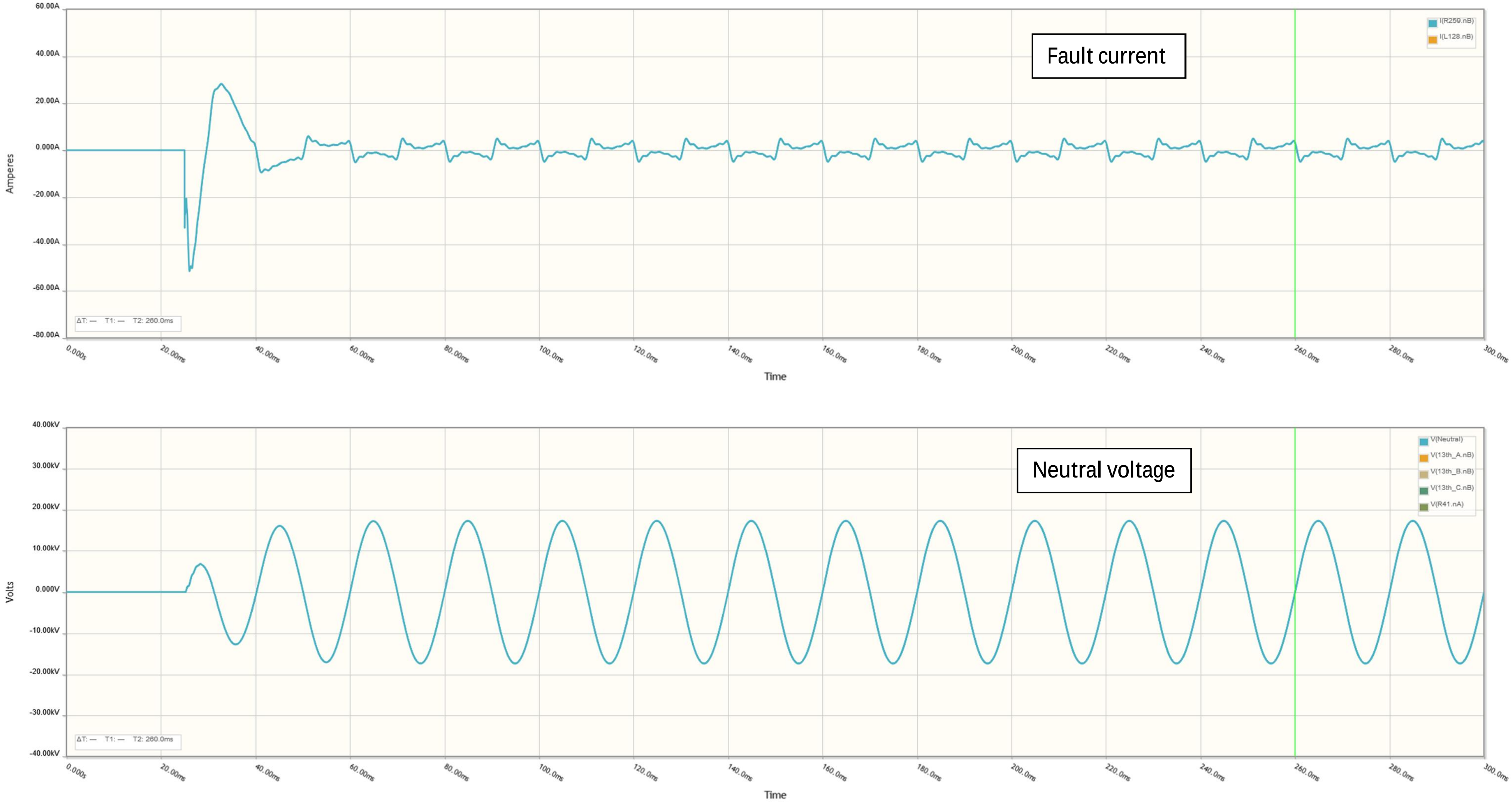
Simulation	Neutral voltage	Fault current
With harmonics	12.28kV	2.407A
Without harmonics	12.28kV	1.536A
Difference	0.0%	57%

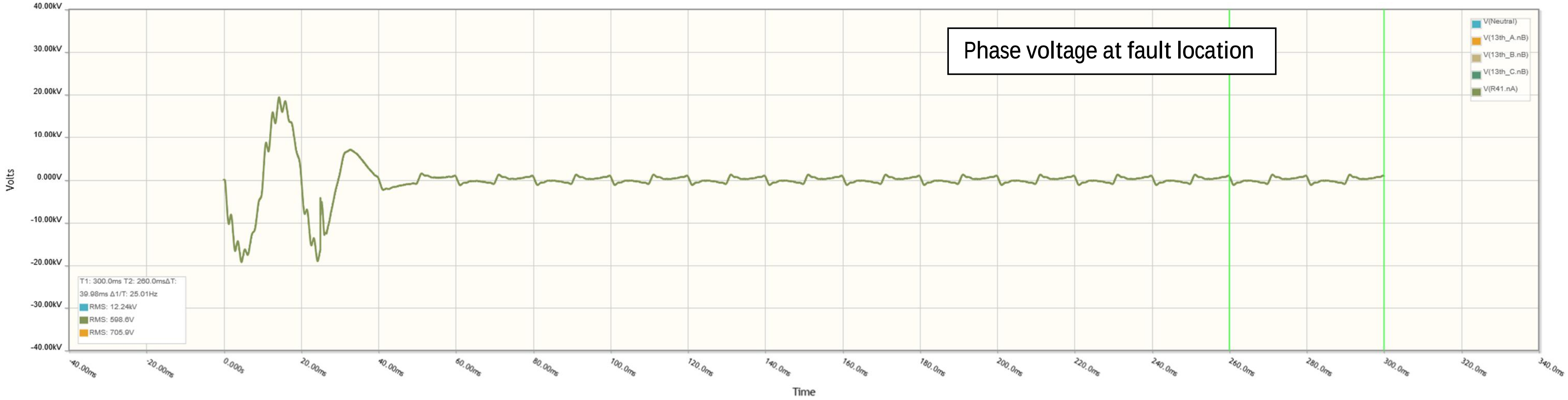
For this low impedance fault:

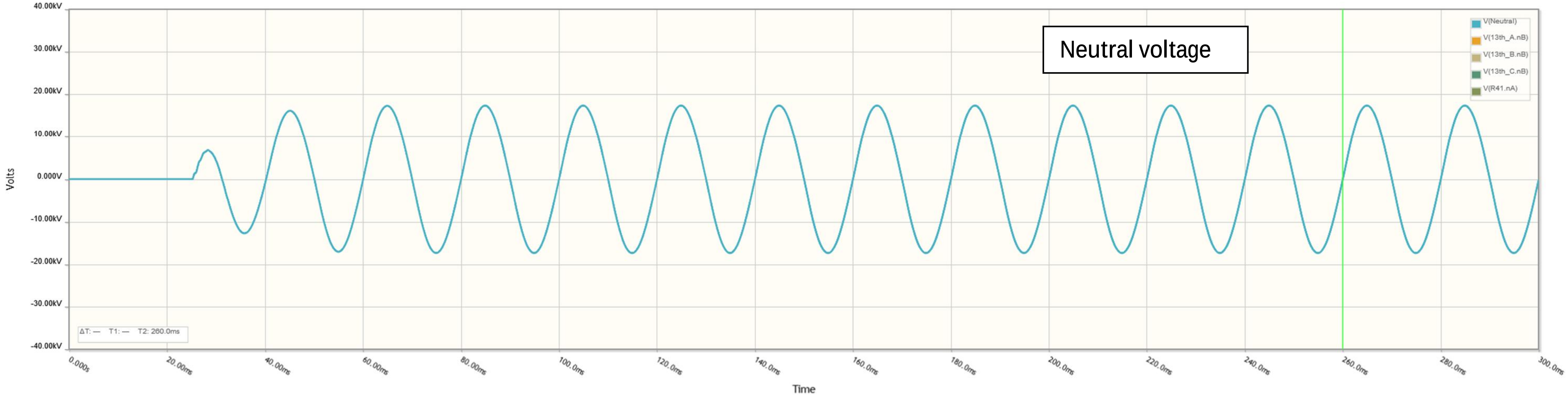
- Harmonics have no effect on neutral displacement voltage (or the ASC coil current)
- Harmonics cause a substantial increase in fault current
- There is a 15% difference between the voltage on the faulted phase at the fault location (601.6 volts) and at the source substation (707.1 volts)

The waveforms produced by the simulation are shown in Figure 30.

Figure 30: low impedance fault with network harmonics – network waveforms

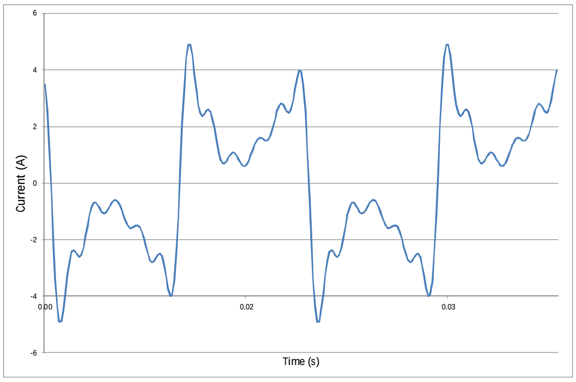


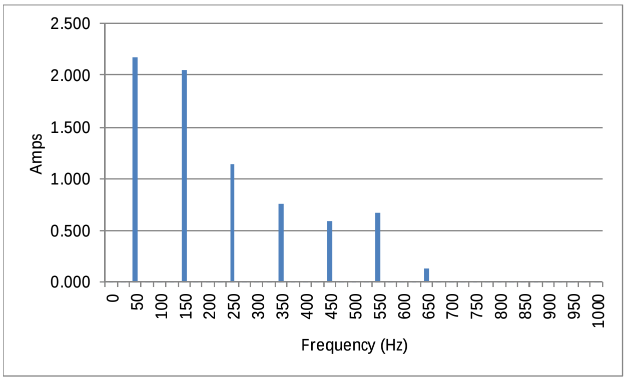

Phase voltage at fault location
Volts
Time
V(Neutral)
V(13th_A.nB)
V(13th_B.nB)
V(13th_C.nB)
V(R41.nA)


Neutral voltage
Volts
Time
V(Neutral)
V(13th_A.nB)
V(13th_B.nB)
V(13th_C.nB)
V(R41.nA)

The fault current waveform and spectrum are shown in Figure 31.

Figure 31: low impedance fault current with harmonics – remote 250 ohm earth fault





The 'one on N' harmonic profile can be seen reflected in the fault current. The 50Hz current has been suppressed by the resonant earthing and the 11th harmonic current is still increased by the network resonance near 632Hz, but the primary reason for the increase in fault current is the decrease in fault impedance. This has increased the amplitude of all harmonic currents flowing into the fault.

For example, the third harmonic component of fault current (2.052A_{pk}) is within 5% of the value of the third harmonic source voltage (539 v_{pk}) divided by a resistance equal to the fault resistance (250Ω) plus the series resistance of the faulted phase between the fault and the source substation (24.5Ω).

Effect of harmonics on a very low impedance fault

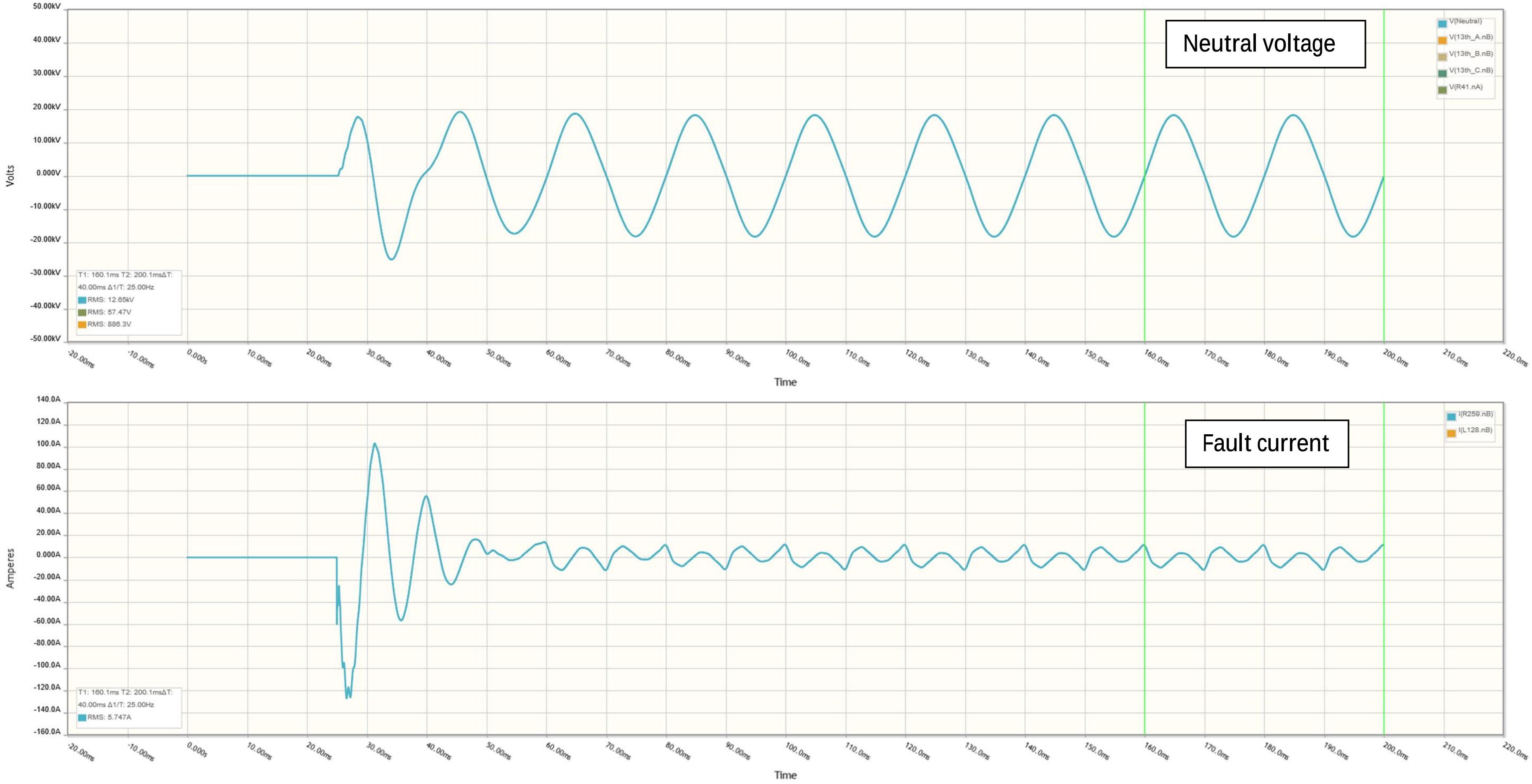
The simulation was repeated with a ten ohm fault resistance at the same location. Given the 3.25 ohm resistance of a 65km earth return path and the typical zone substation earth grid resistance of two ohms, this is approaching the lowest possible earth fault impedance likely to occur in practice.

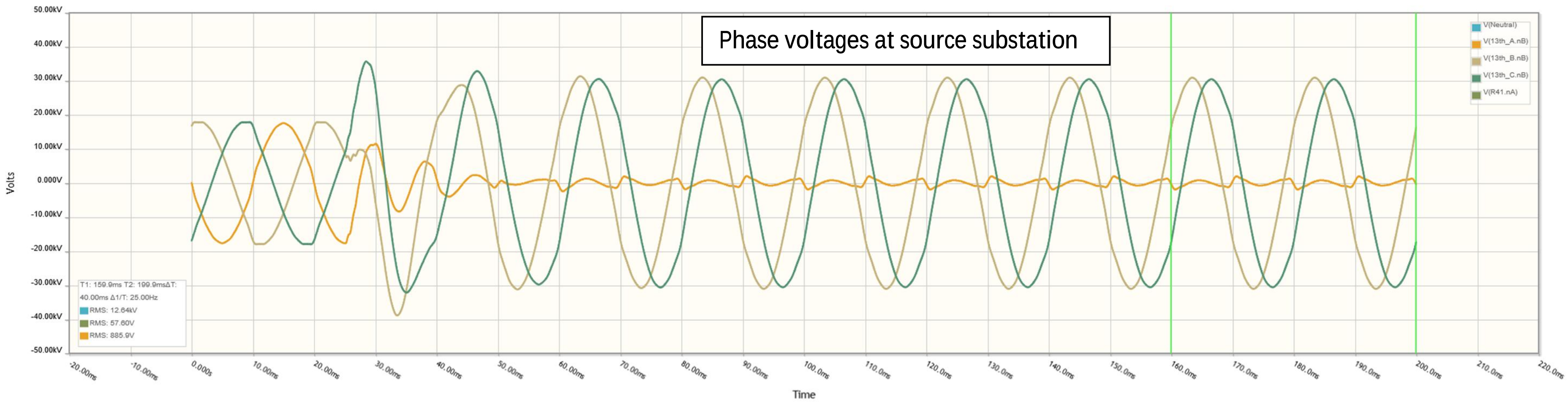
The neutral displacement in the simulation is 99.4% or 12.64kV_{rms}. The fault current is 5.760A_{rms}, most of it harmonic currents. The waveforms are shown in Figure 32 on the next page.

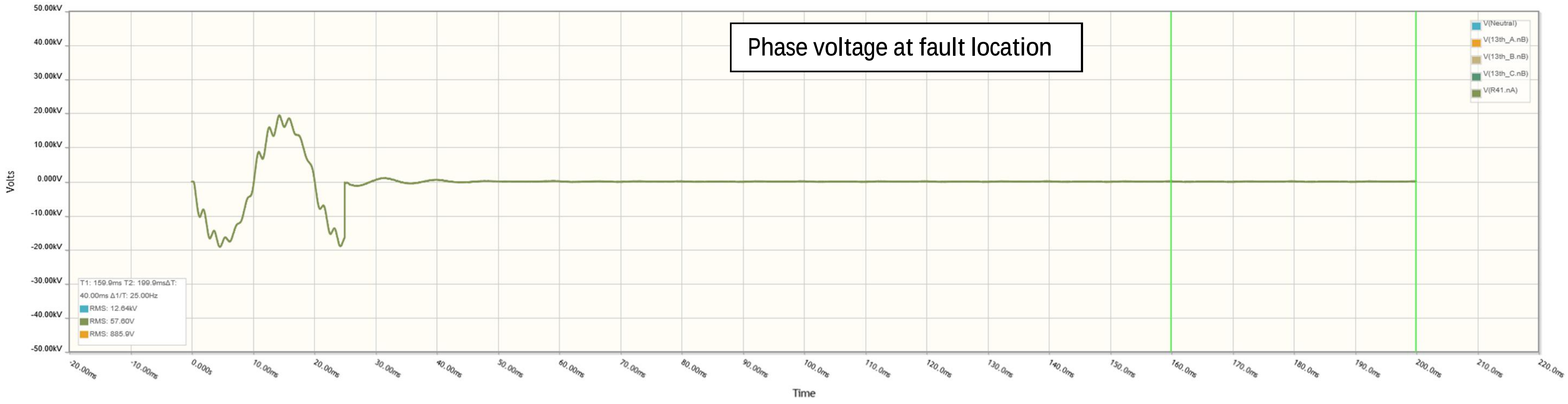
It can be seen that the fault current settles to a steady state harmonic-rich waveform. There is an initial transient comprising a few cycles of a decaying oscillatory current with a frequency of about 120Hz and decay time constant of the order of 10-15ms.

In the first 50Hz cycle after the fault, the neutral voltage peaks at 25.2kV leading to the Phase B voltage reaching 38.8kV. This over-voltage is not due to fast travelling wave effects but to the various resonances in the network model and a significant (decaying) DC component in the initial transient.

Figure 32: very low impedance remote fault with harmonics

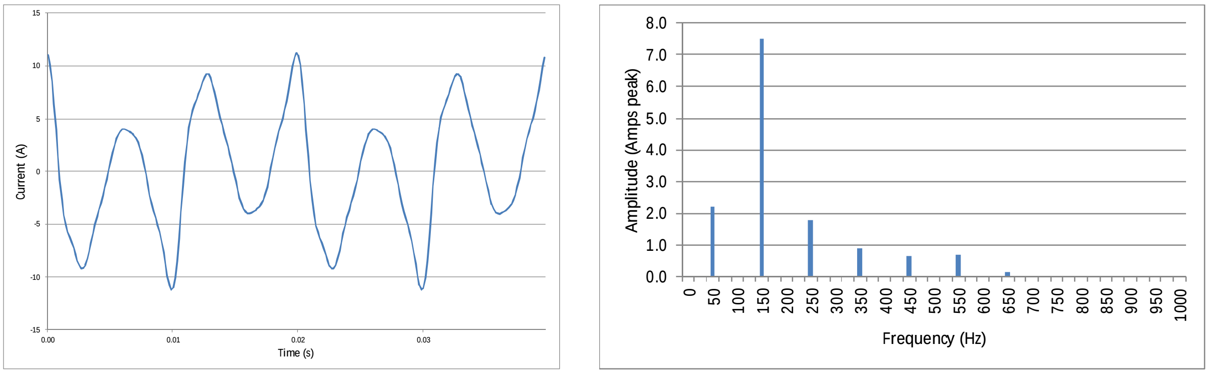



Phase voltages at source substation
Volts
Time
V(Neutral)
V(13th_A.nB)
V(13th_B.nB)
V(13th_C.nB)
V(R41.nA)
T1: 159.9ms T2: 199.9msΔT: 40.00ms Δ1/T: 25.00Hz
RMS: 12.64kV
RMS: 57.60V
RMS: 885.9V


Phase voltage at fault location
Volts
Time
V(Neutral)
V(13th_A.nB)
V(13th_B.nB)
V(13th_C.nB)
V(R41.nA)
T1: 159.9ms T2: 199.9msΔT: 40.00ms Δ1/T: 25.00Hz
RMS: 12.64kV
RMS: 57.60V
RMS: 885.9V

The waveform and spectrum of the residual fault current are shown in Figure 33.

Figure 33: fault current in a very low impedance fault with harmonics – 10 ohm remote earth fault



The third harmonic dominates the fault current. The 50Hz component of fault current (1.578A_{rms}) implies a value for R_n of 8,051 ohms which is similar to the values derived in Table 9. While the fundamental component of fault current has remained at the same level, all harmonic components of the fault current have increased as the fault resistance has reduced. The overall effect is a 260% increase in fault current from 1.6 amps to 5.76 amps.

Overall conclusion – effect of harmonics

The simulation results with and without harmonics are summarised in Table 16.

Table 16: Effect of network harmonics on earth faults 65 km from substation

Fault resistance (ohms)	Fault current (amps)	Neutral voltage (kilovolts)	50Hz current (amps)	150Hz current (amps)	250Hz current (amps)	350Hz current (amps)	450Hz current (amps)	550Hz current (amps)	650Hz current (amps)
3200	1.141	9.060	1.141	-	-	-	-	-	-
3200	1.184	9.057	1.140	0.128	0.090	0.085	0.107	0.229	0.060
250	1.536	12.28	1.536	-	-	-	-	-	-
250	2.407	12.28	1.535	1.451	0.803	0.533	0.418	0.471	0.097
10	1.581	12.64	1.581	-	-	-	-	-	-
10	5.760	12.64	1.578	5.314	1.257	0.647	0.457	0.497	0.102

The investigation into the effects of harmonics leads to the following conclusions (for an accurately tuned, balanced network with total harmonic distortion of 0.08% in network voltages and effects measured on an earth fault located 65 km from the substation):

1. Network harmonics make little difference to the energy in high impedance faults. Harmonics produce only a 3.8% increase in the current that flows in a 3,200 ohm earth fault.
2. The effect of network harmonics on low impedance faults is moderate. Harmonics can increase the current that flows in a 250 ohm earth fault by 57% from 1.5 amps to 2.4 amps.

3. Network harmonics have potentially major effects on very low impedance faults. Harmonics dominate the current that flows in a 10 ohm earth fault, increasing it by 260% from 1.6 amps to 5.75 amps.
4. Resonances in networks can lead to disproportion prominence of some higher order harmonics in earth fault currents.
5. Harmonics have little or no effect on neutral displacement voltage or ASC coil current as these are determined primarily by the 50Hz resonance between the coil and total network capacitance.
6. Each harmonic can be separately calculated and the results added together – this is evident from Table 16 which show the 50Hz component of the fault current in each case is identical with the 50Hz current in the simulation with no harmonics.